Spatiotemporal patterns in cyclic Potts model

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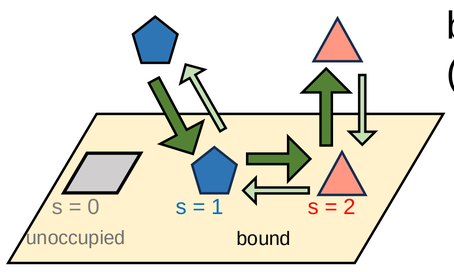
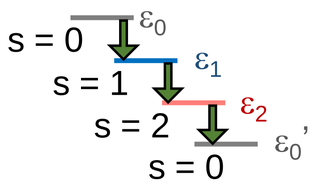
We studied the dynamics of a three-state cyclic Potts model. In symmetric condition, at low cycling energy, homogeneous phases cyclically change (HC) via nucleation and growth, while spiral waves (SW) are formed at high energy. For small and large systems, continuous and discontinuous transitions occur between two modes [1]. In asymmetric conditions, amoeba-like locomotion of biphasic domains and transitions to non-cyclic one-state phases are found [2].

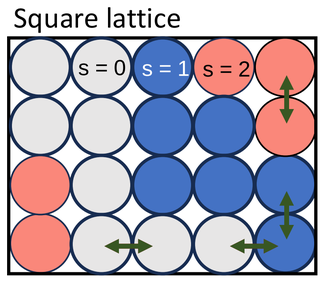
model



Monte Carlo with local detailed-balance condition (s = i → j vs. j → i)



Square lattice



$J = -2k_BT$

Phase separation

$h_{01} = \varepsilon_0 - \varepsilon_1$

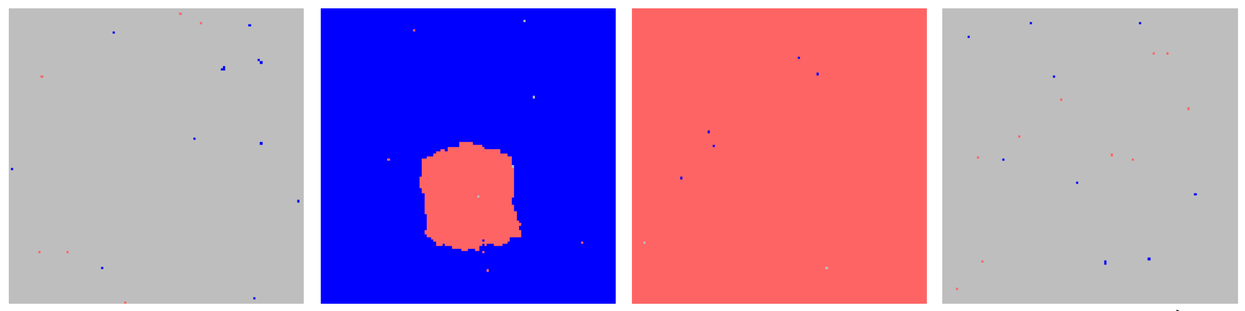
$h_{12} = \varepsilon_1 - \varepsilon_2$

$h_{20} = \varepsilon_2 - \varepsilon_0'$

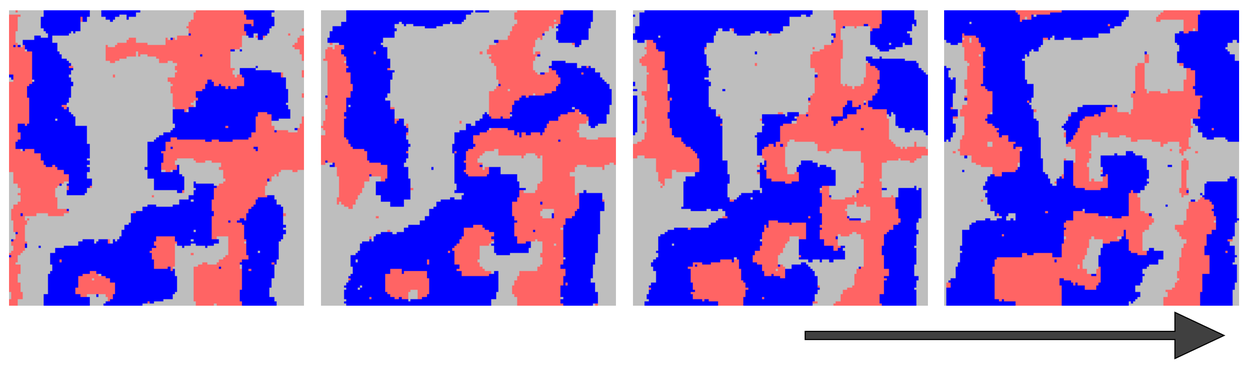
cyclic-symmetry condition

$h_{01} = h_{12} = h_{01} = h$

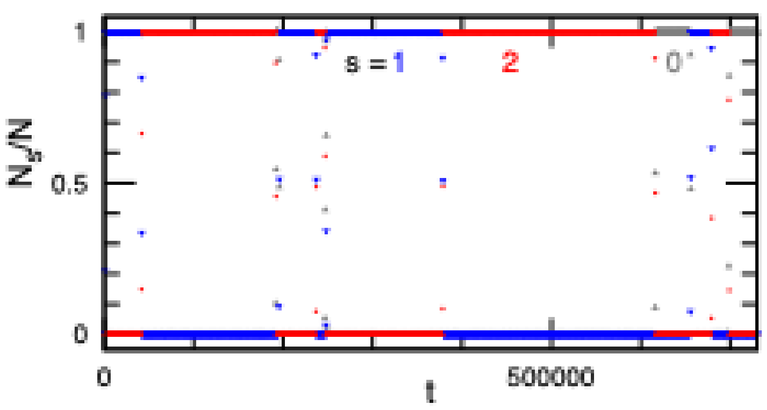
Homogeneous cycling mode (HC)



Spiral wave mode (SW)



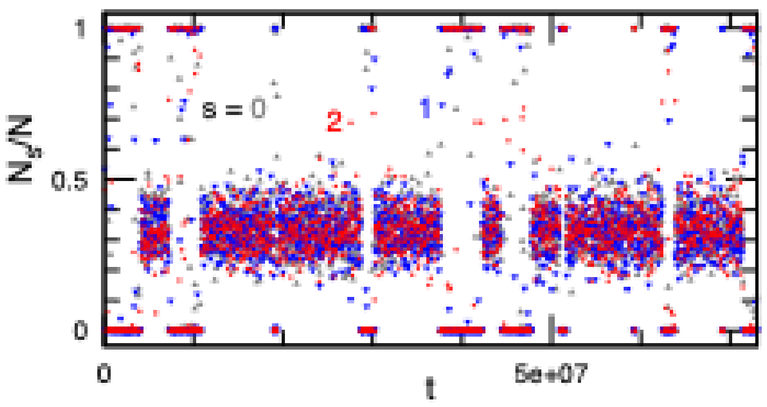
Time evolution at N = 128²



cyclic-symmetry condition

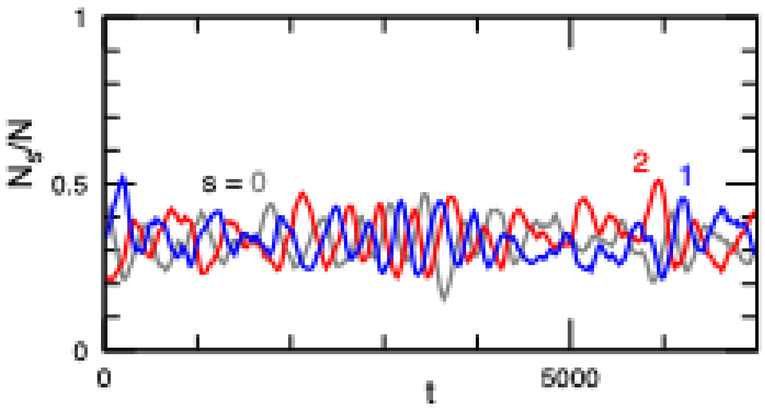
$h/k_BT = 0.9$

HC



$h/k_BT = 1.1$

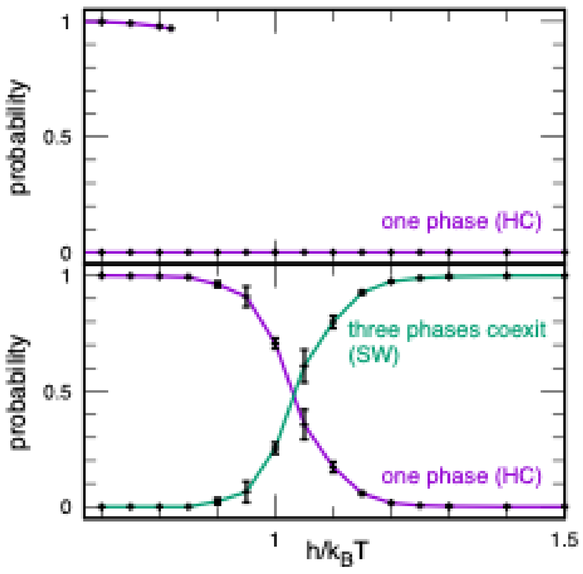
Coexistence
HC + SW



$h/k_BT = 1.5$

SW

Ratio of Two Modes



$N = 256^2$

Discontinuous

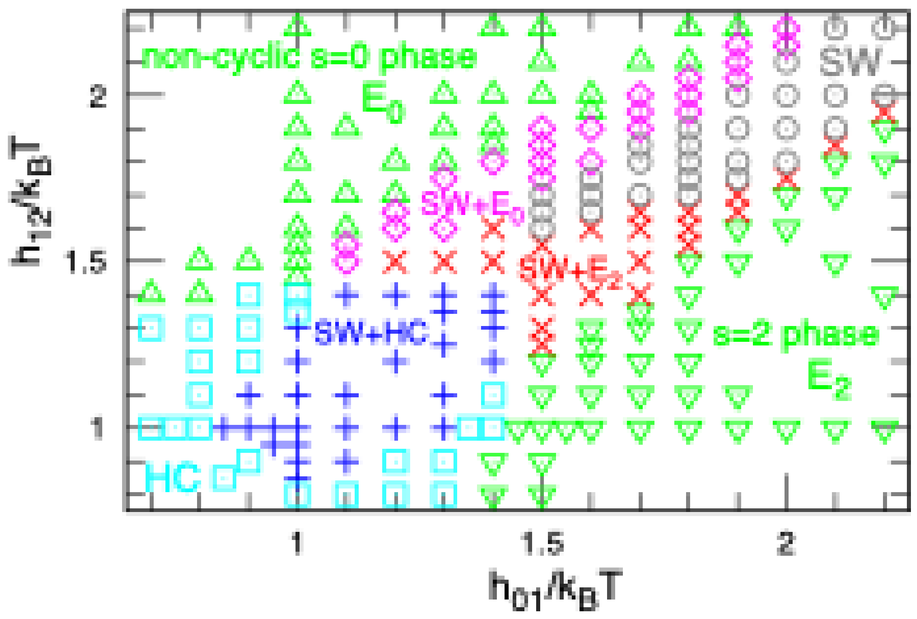
$N = 128^2$

Continuous

Nucleation vs. Growth
SW occurs when new nucleation starts during the growth.

Phase diagram

$h_{20}/k_BT = 1$ and $N = 128^2$



An increase in h_{01} induces formation of non-cyclic s = 2 phase, owing to suppression of nucleation of s = 0 domains.

[1] H. Noguchi, F. van Wijland, and J.-B. Fournier, J. Chem. Phys. 161, 025101 (2024).
[2] H. Noguchi and J.-B. Fournier, New J. Phys. 26, 093043 (2024).