

**Criteria for the validity of Amontons Coulomb's law;  
Study of friction  
using dynamics of driven vortices of superconductor**

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# Outline

## 1) Background:

Problems in physics of friction  
Collective dynamics of quantum condensate  
Dynamics of driven vortices in superconductors

## 2) Physics of friction by using dynamics of driven vortices

Model

Kinetic friction as a function of velocity

Broadened dynamical phase transition

**A. Maeda *et al.*: Phys. Rev. Lett. 94 (2005) 077001.**

Static friction as a function of waiting time

Criteria for the validity of Amontons-Coulomb friction

**D. Nakamura *et al.*: arXiv 0906.3086.**

## 3) Response to ac ( $\mu$ -wave) driving force

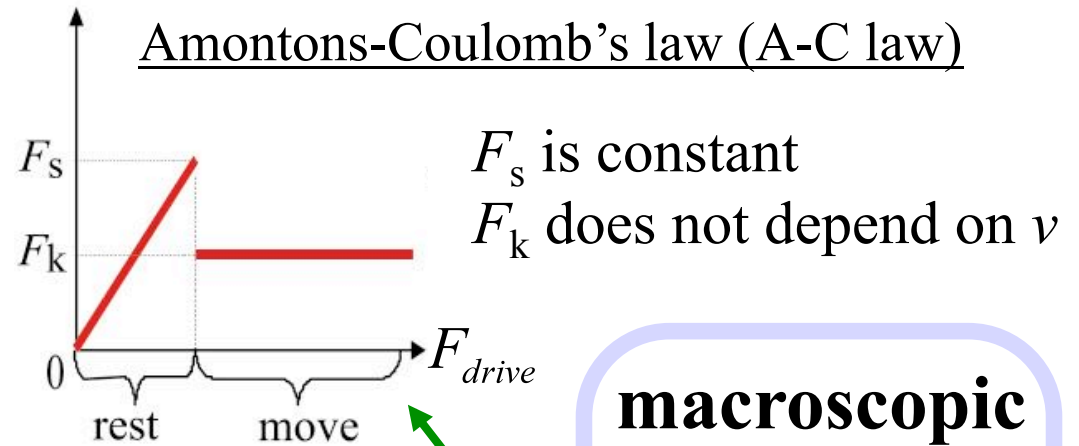
## 4) Conclusion, perspective



# Microscopic friction vs Macroscopic friction

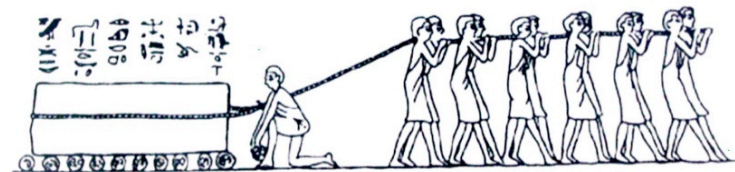
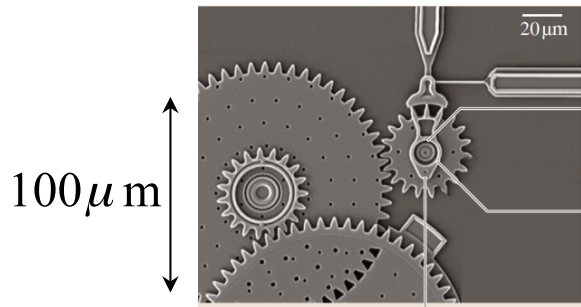
$F_s$  : maximum static friction force

$F_k$  : kinetic friction force



**microscopic friction**

**macroscopic friction**



*In some cases, even the macroscopic friction does not obey the A-C law*

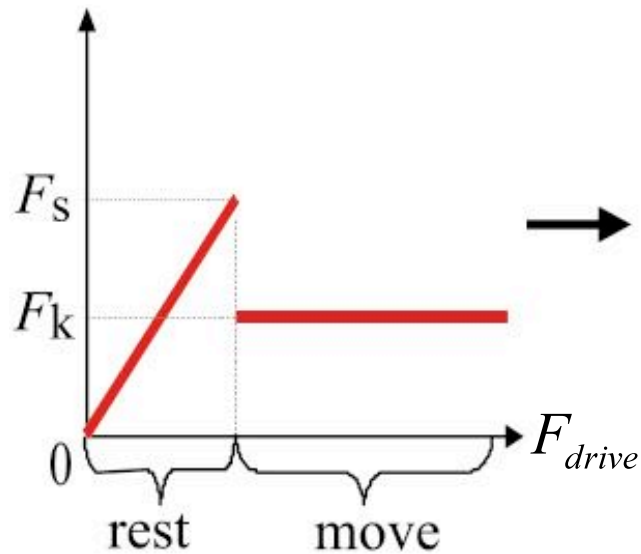
D. M. Tanner *et al.*, *MEMS Reliability : Infrastructure, Test Structures, Experiments, and Failure Modes* 2000-2091 (Sandia National Laboratories, 2000).

**What is the limit of the validity of Amontons-Coulomb's law?**

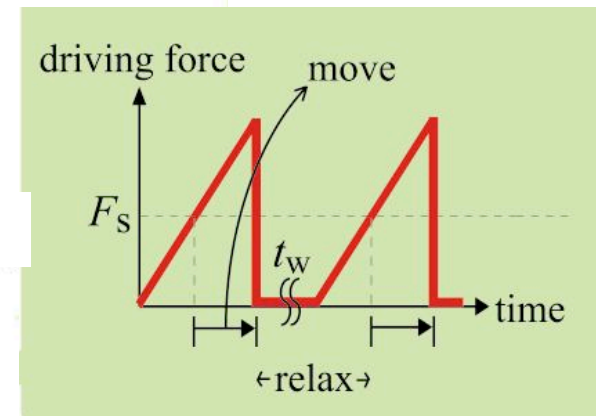
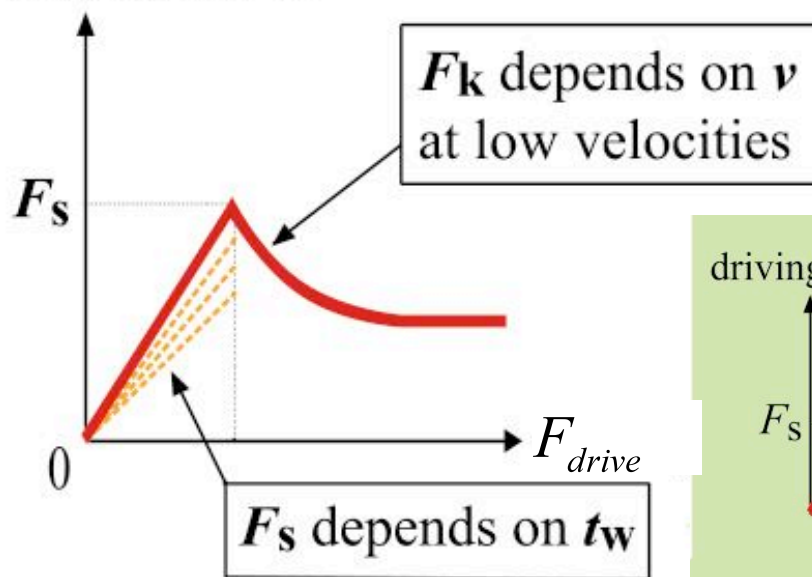
a)

# Amontons-Coulomb friction vs real friction

Amontons-Coulomb's law



Real friction system



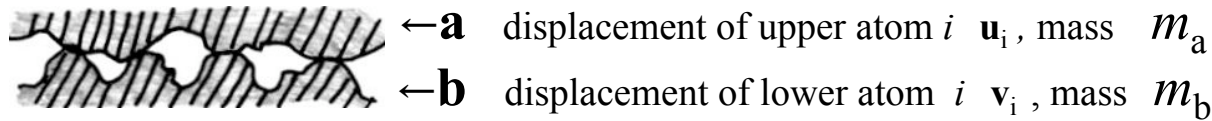
To understand friction from microscopic point of view\_

Need model system with which systematic experiments are possible in a repeated manner (free from wear)



# Microscopic formulation of friction

H. Matsukawa and H. Fukuyama:  
PRB 49, 17286 (1994)

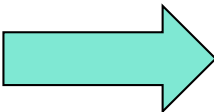


$$m_a \ddot{\mathbf{u}}_i + m_a r_a (\dot{\mathbf{u}}_i - \langle \dot{\mathbf{u}}_i \rangle_i) = \sum_{j \in \mathbf{a}} \mathbf{F}_a(\mathbf{u}_i - \mathbf{u}_j) + \sum_{j \in \mathbf{b}} \mathbf{F}_I^{(i,j)}(\mathbf{u}_i - \mathbf{v}_j) + \mathbf{F}_{\text{ex}} + \mathbf{F}_G \quad \text{eq. motion for an upper atom } i$$

$$m_b \ddot{\mathbf{v}}_i + m_b r_b (\dot{\mathbf{v}}_i - \langle \dot{\mathbf{v}}_i \rangle_i) = \sum_{j \in \mathbf{b}} \mathbf{F}_b(\mathbf{v}_i - \mathbf{v}_j) + \sum_{j \in \mathbf{a}} \mathbf{F}_I^{(i,j)}(\mathbf{v}_i - \mathbf{u}_j) + \mathbf{F}_S(\mathbf{v}_i) + \mathbf{F}_G \quad \text{eq. motion for a lower atom } i$$

dissipation from a representative DF to others

steady state  
 summing up for all atoms  
 time averaged



$$-\sum_{i \in \mathbf{a}} \sum_{j \in \mathbf{b}} \langle F_{I//}(\mathbf{u}_i - \mathbf{v}_j) \rangle_t = N_a \langle F_{\text{ex}} \rangle_t$$

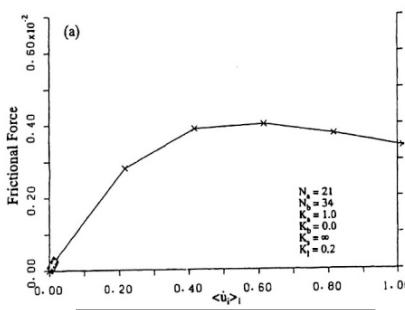
friction: sum of interatomic (pinning) forces

## 1 D model for clean surfaces

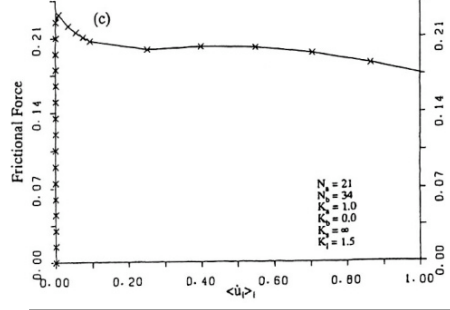
numerical solution for the above equation

- **clean surface**  
 finite  $F_k$  even for zero  $F_s$
- **disordered surface**  
 less velocity dependent  
 similar to Amontons-Coulomb's law

## $F_k$ as a function of velocity



clean surface



(normal) dirty surface



# Model systems for friction study in quantum condensate in solids

## Charge-density wave (CDW)

1D

$$m \ddot{\mathbf{u}}_i + m\gamma \dot{\mathbf{u}}_i + \sum_{i,j} \mathbf{F}(\mathbf{u}_i - \mathbf{u}_j) + \sum_i \mathbf{F}_p^{(i)}(\mathbf{u}_i) = \mathbf{F}_{ex}$$

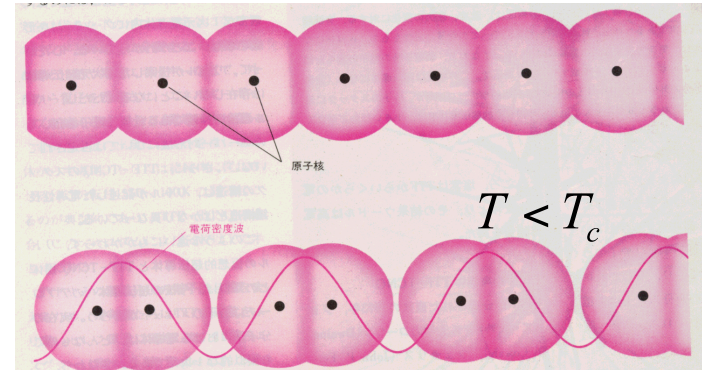
$$\mathbf{F}_{ex} = -e\mathbf{E}$$

$$\mathbf{j} = \sum_i -e \dot{\mathbf{u}}_i$$

$\mathbf{u}_i$  : displacement of  $i$ -th electron in the CDW

$m$ : mass of the  $i$ -th electron

$F_p$ : pinning force for  $i$ -th electron



## Vortex lattice of superconductor

2D

$$(m \ddot{\mathbf{u}}_i + \gamma \dot{\mathbf{u}}_i) + \sum_{i,j} \mathbf{F}(\mathbf{u}_i - \mathbf{u}_j) + \sum_i \mathbf{F}_p^{(i)}(\mathbf{u}_i) = \mathbf{F}_{ex}$$

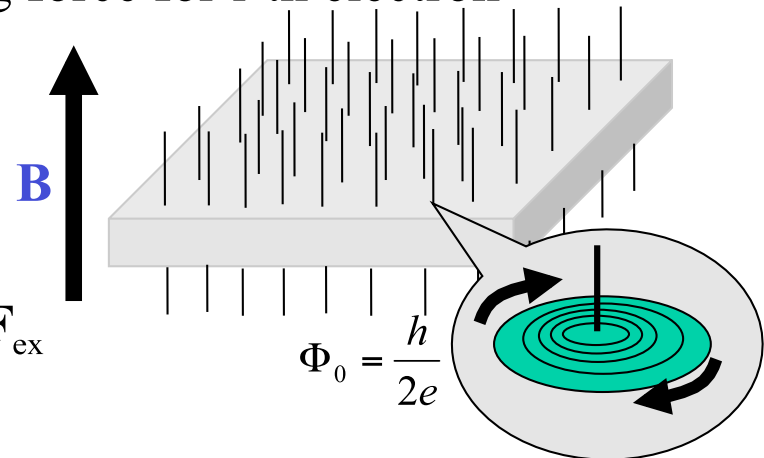
$$\mathbf{F}_{ex} = \Phi_0 \times \mathbf{j}$$

$$\mathbf{E} = \sum_i \Phi_0 \times \dot{\mathbf{u}}_i$$

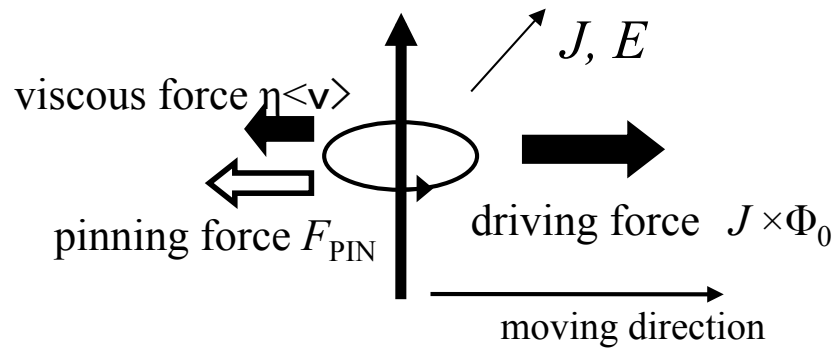
$\mathbf{u}_i$  : displacement of  $i$ -th vortex in the lattice

$m$ : mass of the  $i$ -th vortex in the lattice

$F_p$ : pinning force for  $i$ -th vortex



# Vortex dynamics - the ideal model system of friction



$$\kappa_P u + \eta \dot{u} = J \times \Phi_0 + f_{\text{fluct}}$$

elastic pinning      flux flow  
creep motion

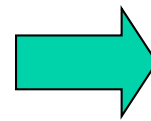
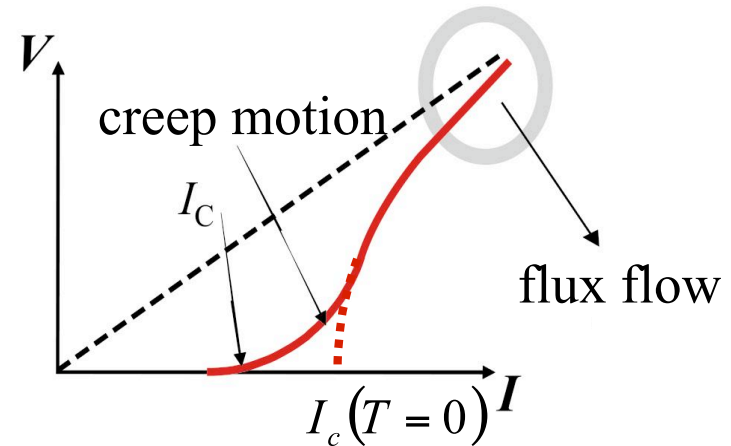
Friction force on solid-solid interface

$$-\sum_{i \in a} \sum_{j \in b} \langle F_{I//}(\mathbf{u}_i - \mathbf{v}_j) \rangle_t = N_a \langle F_{\text{ex}} \rangle_t$$

H. Matsukawa and H. Fukuyama PRB 49, 17286 (1994).

**Common feature**

- multi-internal-degrees of freedom
- non-equilibrium
- non-linearity



$$F_s = j_c \times \Phi_0$$

$$F_k = j \Phi_0 - \eta v$$

A. Maeda *et al.*, PRL 94, 077001 (2005).

**Advantage**

- scan  $H$ ,  $T$  and  $F$  continuously
- intrinsically reproducible
- no wear, no debris



## Purpose of study

### Driven vortices as a model system of solid-solid friction

- (1) Measure kinetic friction as a function of velocity
- (2) Static friction as a function of waiting time

Change temperature, magnetic field, pinning, system size

- (3) Find the criteria for the validity of Amontons-Coulomb friction
- (4) Ac dynamics ( $\mu$ -wave to THz)

Microscopic understanding of elementary excitation  
at the interface

High- $T_c$  cuprate superconductor: variety of  $H$ - $T$  phase diagram





## Expressing solid-solid friction in terms of vortex motion

$$F_k = \kappa_P \mathbf{u} = J \times \Phi_0 - \eta \dot{\mathbf{u}}$$

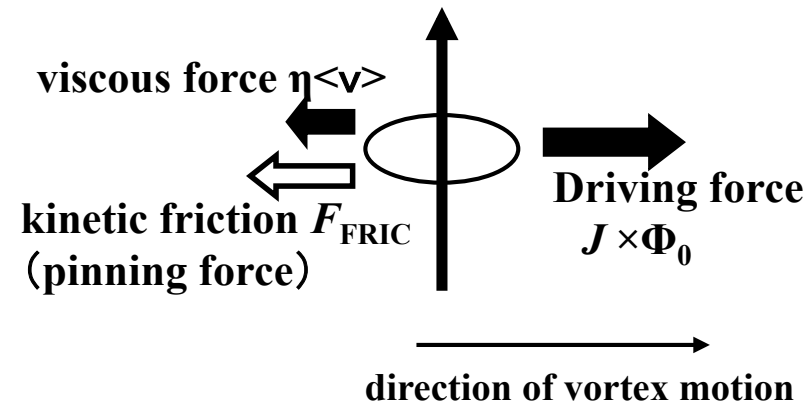
$$= J \times \Phi_0 \left( 1 - \frac{\rho}{\rho(\omega \rightarrow \infty)} \right)$$

$\Phi_0$ : flux quantum  
 $J$ : current density  
 $\rho$ : resistivity

$$\rho(\omega \rightarrow \infty) = \frac{B\Phi_0}{\eta}$$

**Flux flow resistivity**

necessary to make correspondence with theory



$I$ - $V$  measurement and **viscosity,  $\eta$** , measurement can deduce kinetic friction



# Sample preparation

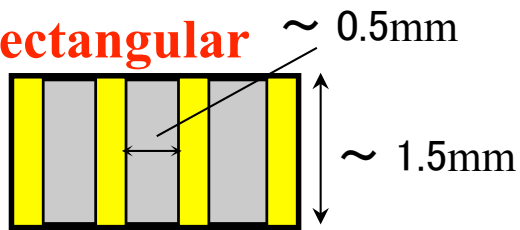
**$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  (LSCO)**  
( $x = 0.12, 0.15$ )

thickness  $\sim 3000 \text{ \AA}$

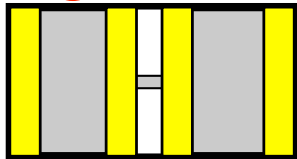
single crystal, thin film

## Fabrication of samples

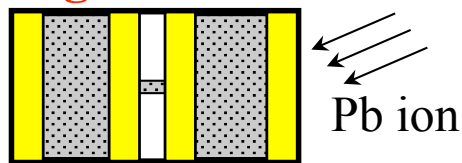
**Rectangular**



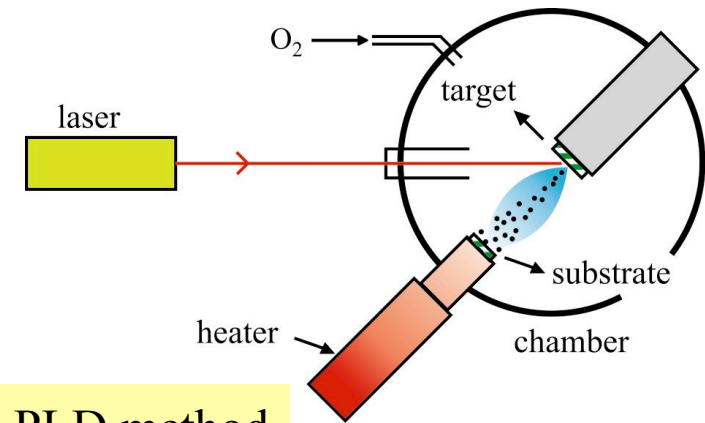
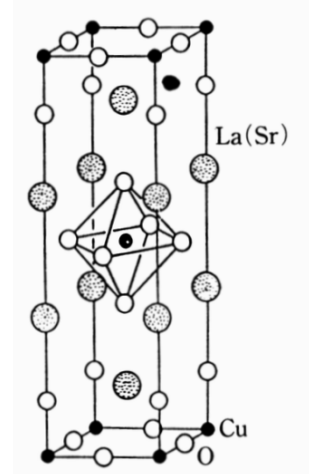
**Bridge**



**Bridge with columnar defects**



## Crystal structure



PLD method

photolithography + chemical etching

S. Komiyama (Univ. of Tokyo)

5.8 GeV Pb ion for the columnar defects

@ Grand Accelérateur National d'Ions Lourds (GANIL)

Ecole Polytechnique, M. Konczykowski and C. J. van der Beek

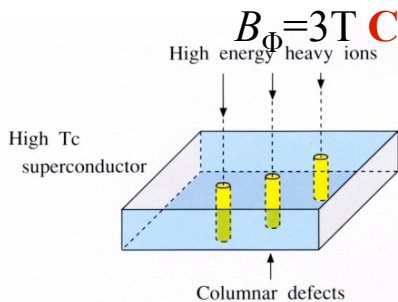


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$F_k(v)$

$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$   
(up to  $\sim 1$  km/s)

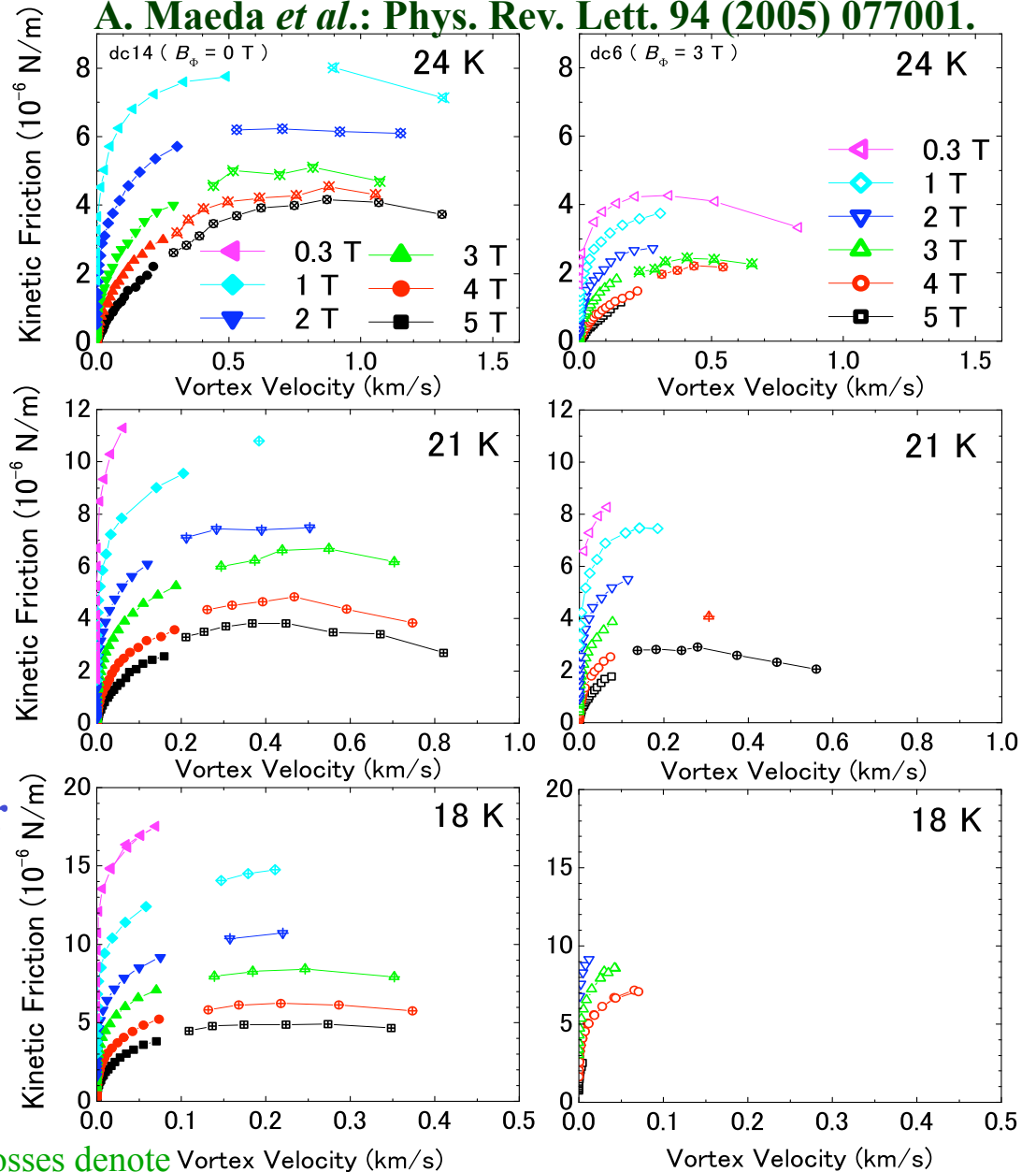
- 1)  $F_k$  changes with  $B$  and  $T$  in a reproducible manner  
good as a model system
- 2) very much different from the Amontons-Coulomb behavior  
similar to "clean surface"
- 3)  $F_k$  saturates and decreases  
existence of a peak in  $F_k(v)$
- 4) smaller  $F_k$  in irradiated samples  
inconsistent with the behavior at low velocities ?



200 MeV Iodine

Data points with crosses denote pulsed measurements

A. Maeda *et al.*: Phys. Rev. Lett. 94 (2005) 077001.

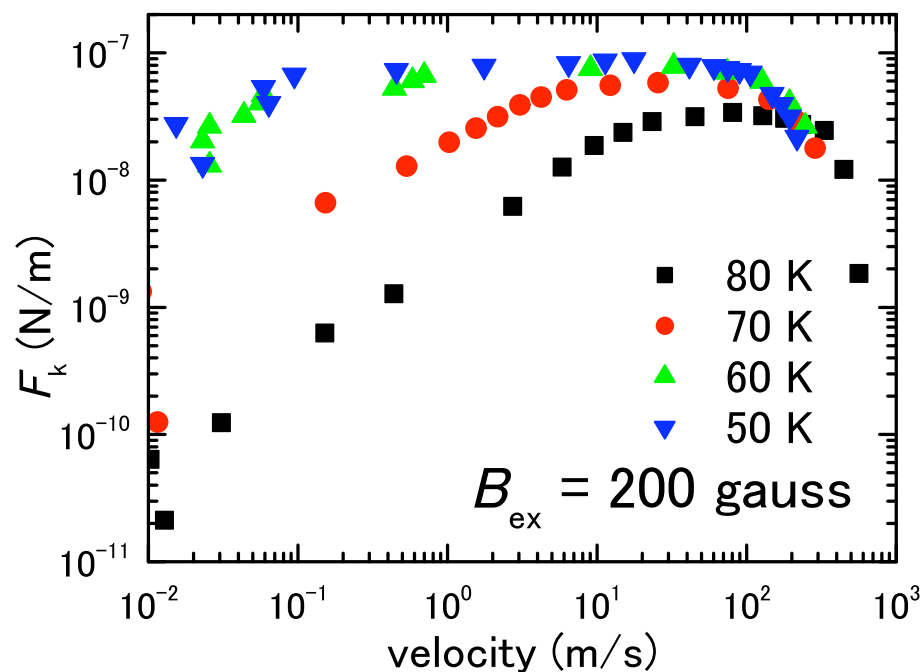
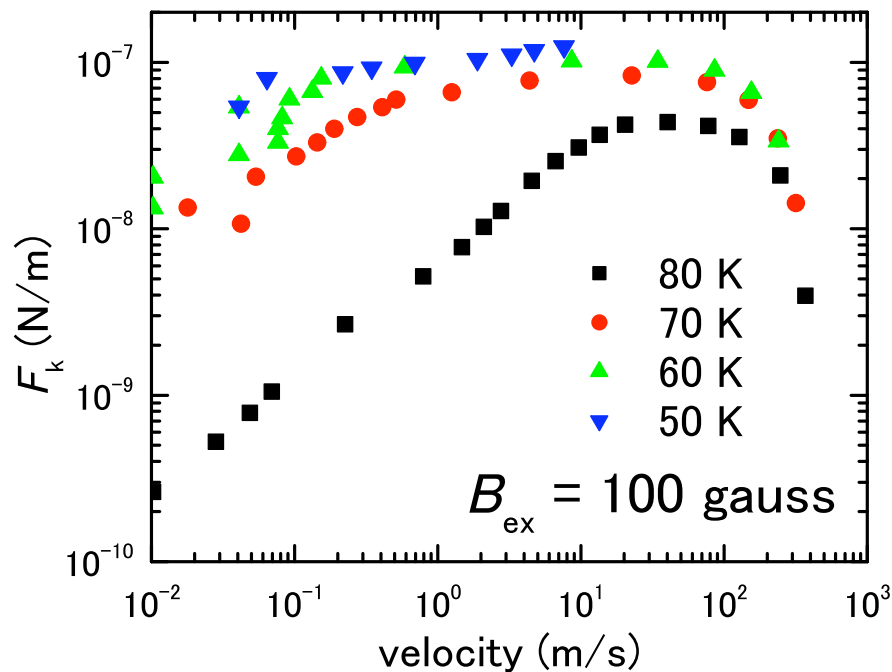


pristine

3T irradiated

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# Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>y</sub> (bulk single crystals)



$$F_k = \phi_0 \times \left( j - \frac{\rho}{\rho_\infty} \right), \quad \rho_\infty = \frac{B\phi_0}{\eta}$$

Universal to many SCs



## Minimal model to explain the data : overdamped equation of motion

S. Savel'ev and F. Nori

$$\eta \dot{x}_i = -\frac{\partial}{\partial x_i} \left( U(x_i) + \sum_j W(x_i - x_j) \right) + F_d + \sqrt{2k_B T} \xi(t)$$

$x_i$  : position of vortices

$\eta$  : viscosity of vortices

$U(x_i)$  : substrate pinning potential

$W(x_i - x_j)$  : inter-vortex interaction

$F_d$  : driving force

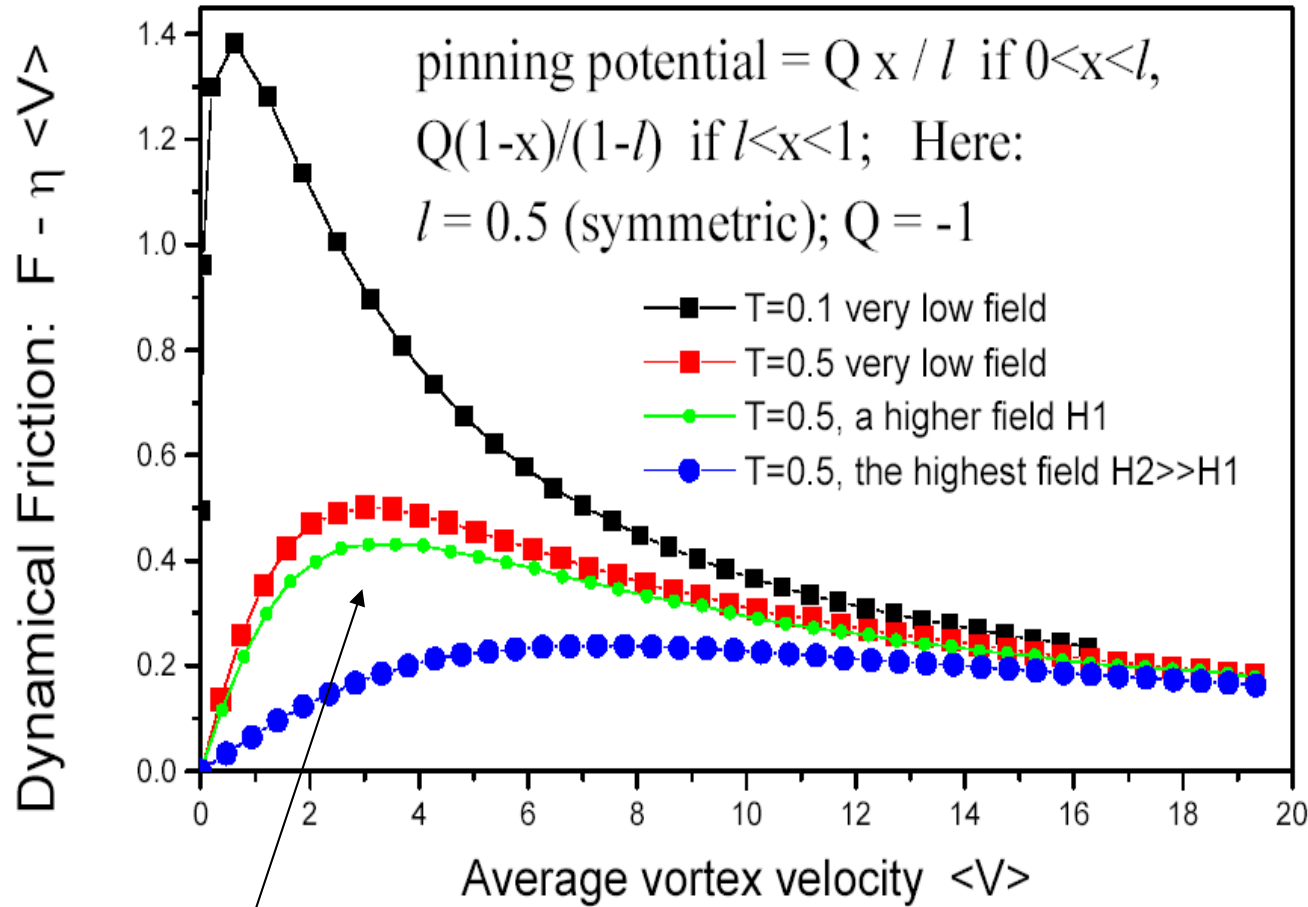
$\xi(t)$  : thermal random force

$T$  : temperature



Numerical simulation for 1D vortex array **at finite temperatures**

S. Savel'ev and F. Nori



$$F_{viscous} \approx F_{pin}$$

a peak  $\therefore \eta v \approx j_c \Phi_0$



## A peak in the kinetic friction $F_k(v)$

velocity at the peak  $v_p \approx \frac{j_c \Phi_0}{\eta}$  **S. Savel'ev and F. Nori**

$$j_c \approx 10^6 \text{ A/cm}^2$$

$$\Phi_0 = 2.07 \times 10^{-7} \text{ gauss} \cdot \text{cm}^2$$

$$\eta \approx 10^{-7} \text{ Ns/m}$$

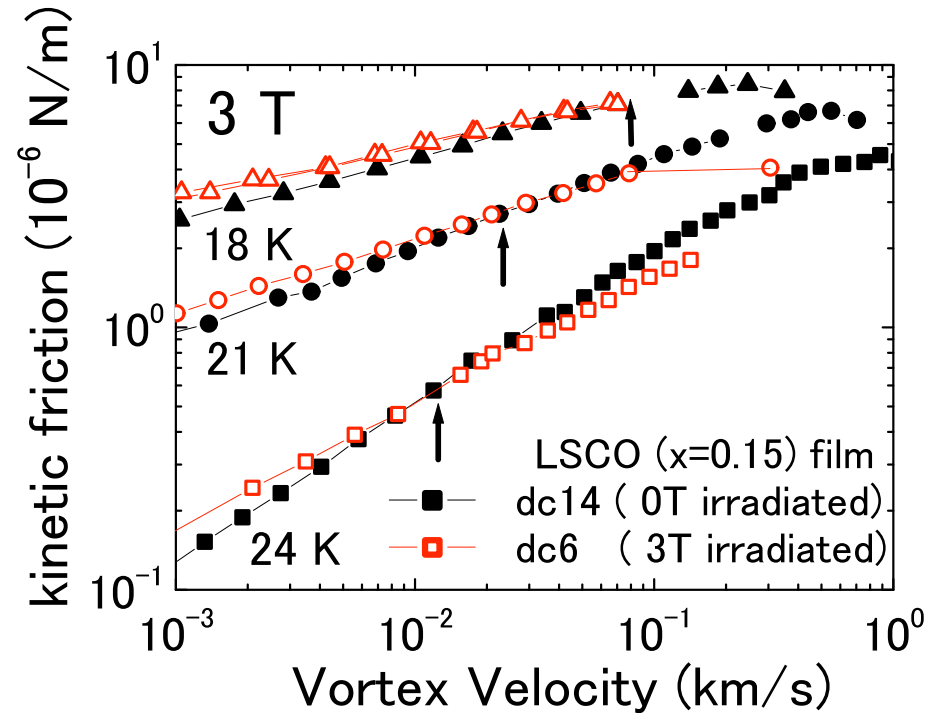
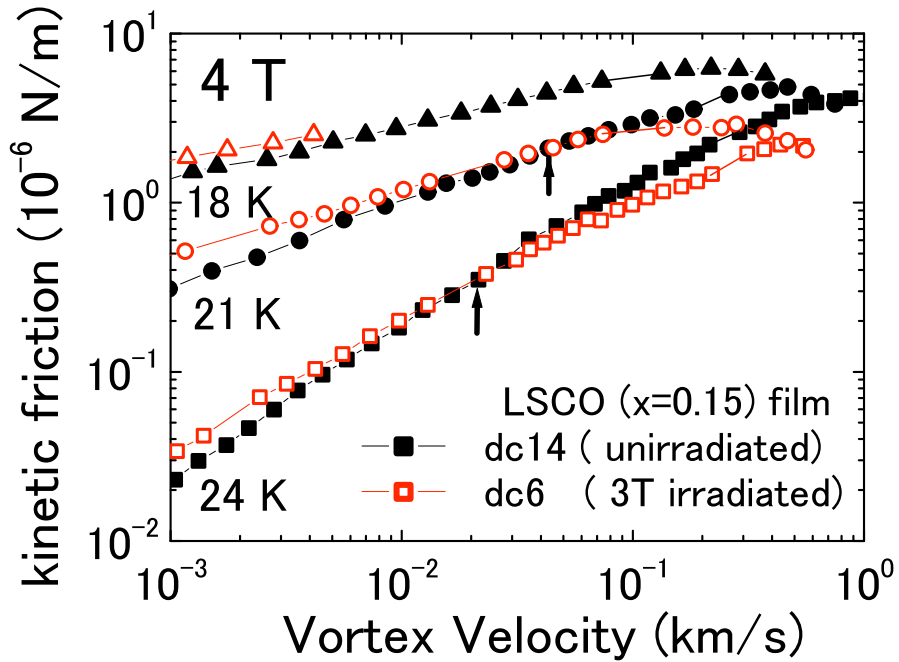
$$v_p \approx 2 \times 10^2 \text{ m/s}$$

**in good agreement with experiment**

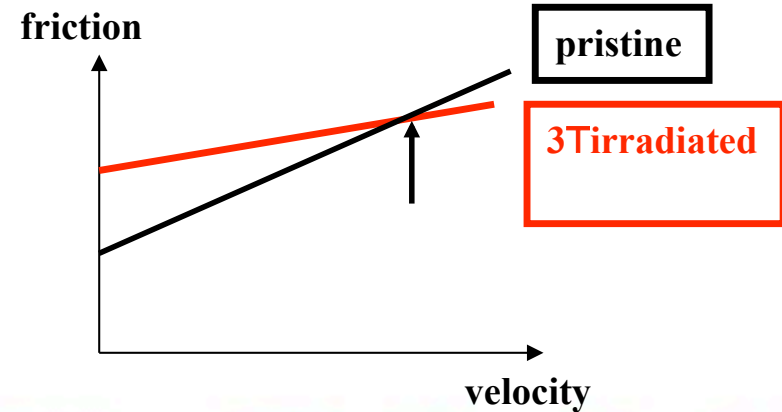
Potential energy plays an important role for  $F_k(v)$ .



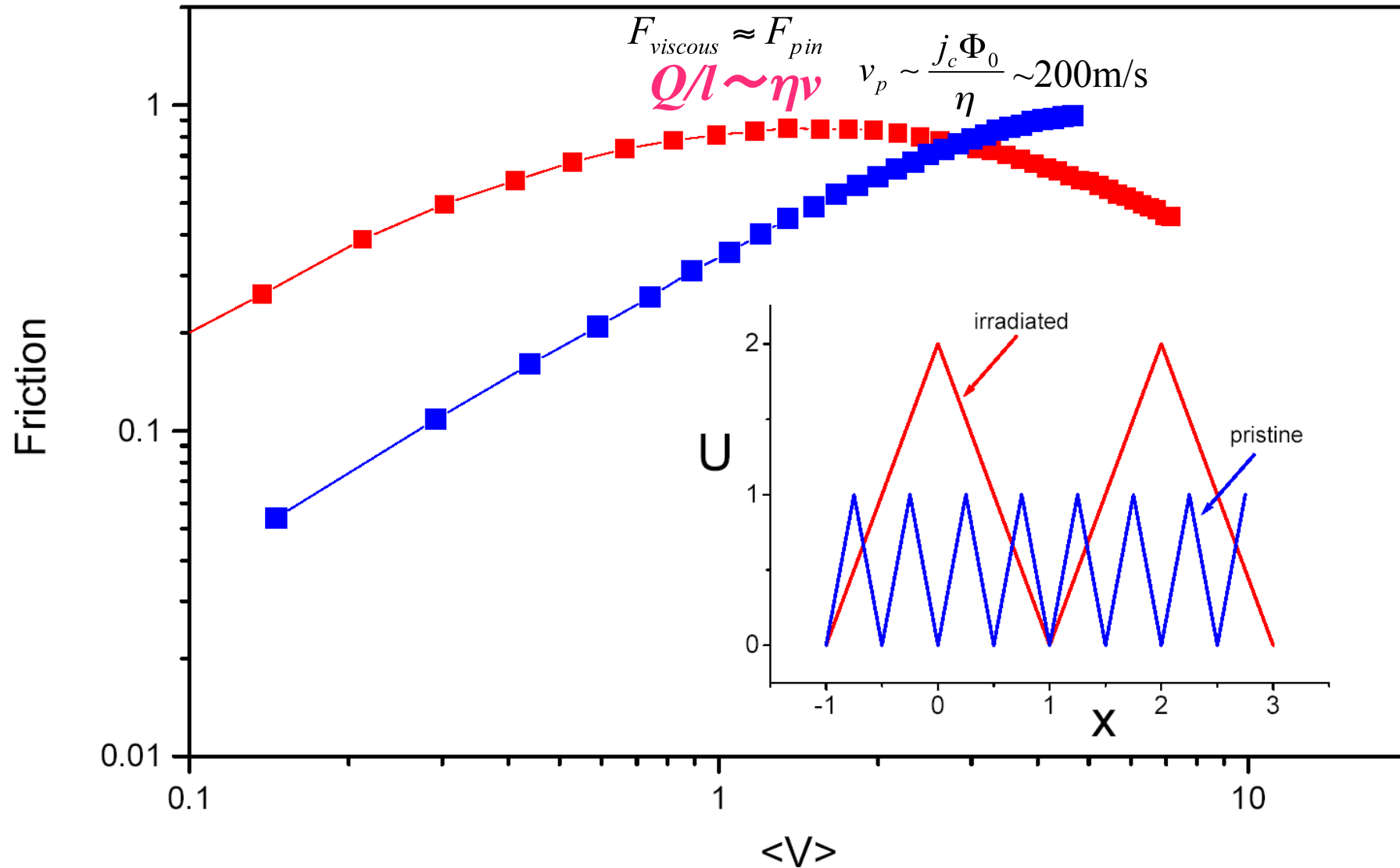
“Inversion” of kinetic friction at intermediate velocities !



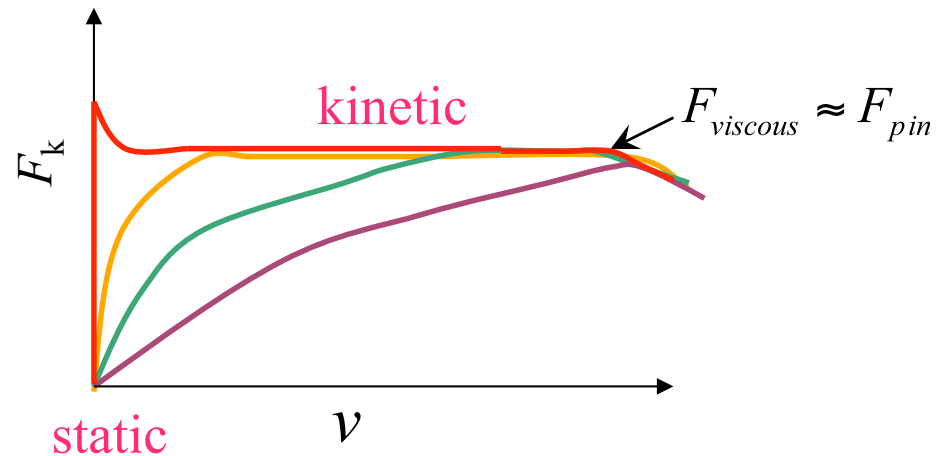
sample with strong pinning  
 higher static friction  
 lower kinetic friction  
 more gradual dependence on  $v$







## Physical origin of the peak



changing parameters

change transition between  
static and kinetic regime

**broaden the transition**

increasing magnetic field

increasing temperature

decreasing system size (macro to micro)



## $N$ strongly coupled system

collective coordinate

$$x_{macro} = \sum_i \frac{x_i}{N}$$

new stochastic variable

$$\xi_{macro} = \sum_i \frac{\xi_i}{N}$$

effective temperature

$$T_{eff} = \frac{T}{N}$$

$$T_{eff} \propto \frac{T}{L^3}$$

( $L$  : system size)

Suggesting the importance of thermal fluctuation in microscopic friction

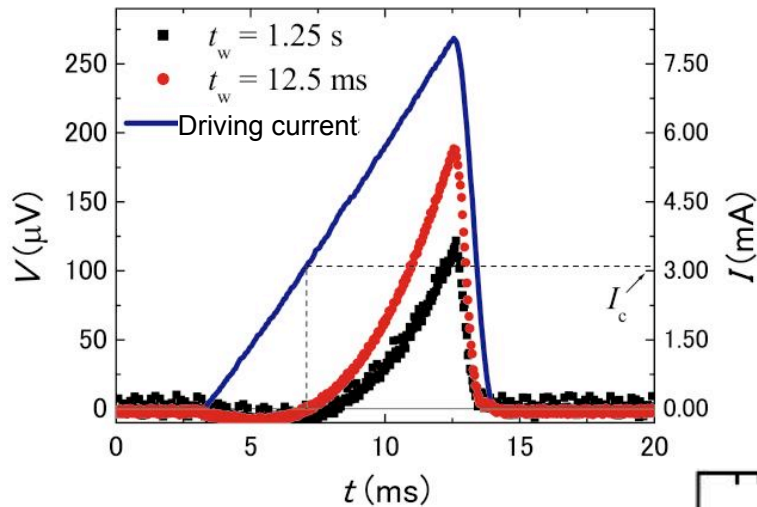




# Measurement of $F_s(t_w)$

D. Nakamura *et al.*: arXiv 0906.3086.

## Rectangular-type thin film

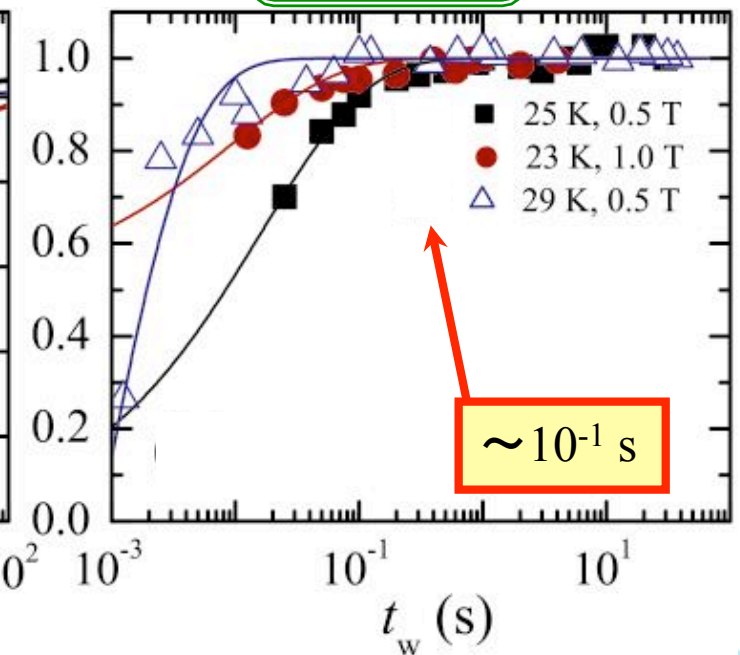
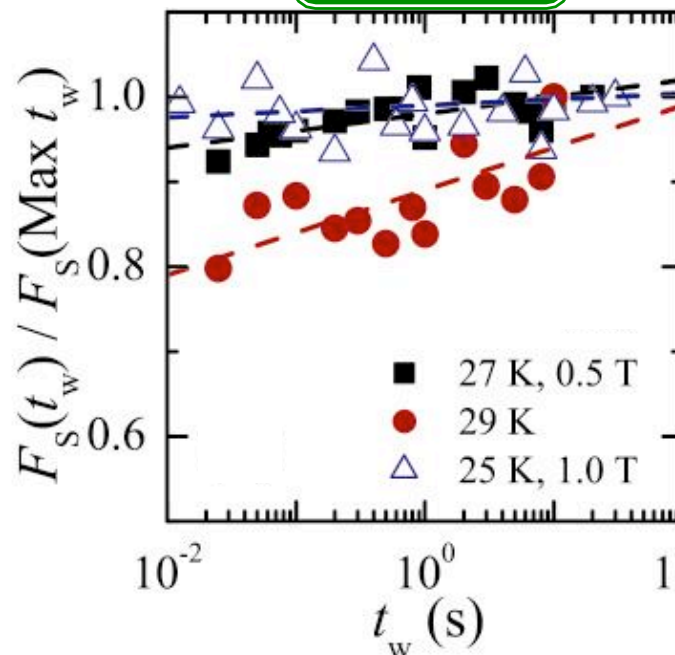
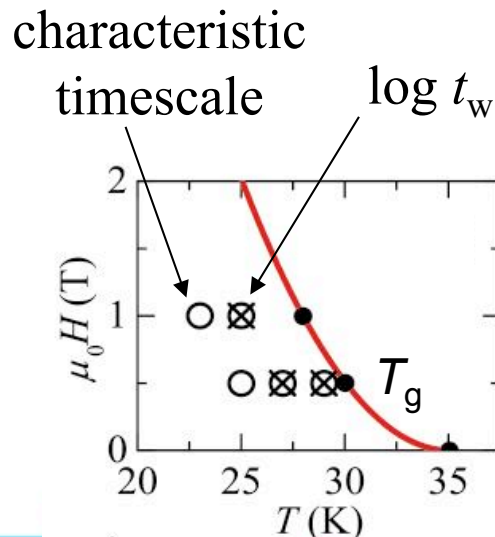


	$x$	$T_c$ (K)
# rectangular A	0.12	35.20
# rectangular B	0.15	35.06

Waveforms become different with  $t_w$   
 $I_c$  depends on  $T \rightarrow$  normalize by  $I_c(\max t_w)$

**High  $T$**

**Low  $T$**



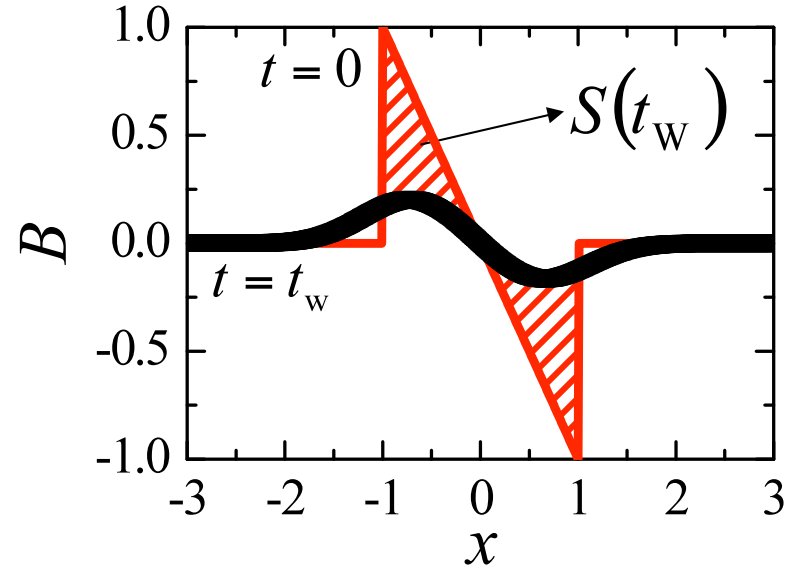
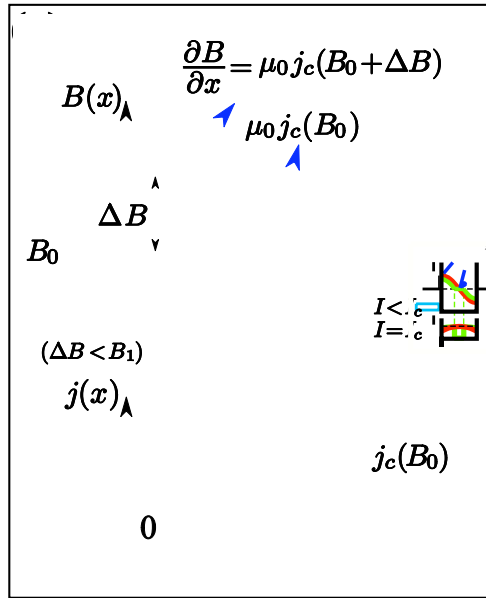
**Logarithmic**

**Characteristic timescale**

# The origin of logarithmic dependence near $T_g$

## Thermally Assisted Flux Flow

P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. **36**, 39(1964).  
 P. H. Kes *et al.*, Supercond. Sci. Technol. **1**, 242(1989).

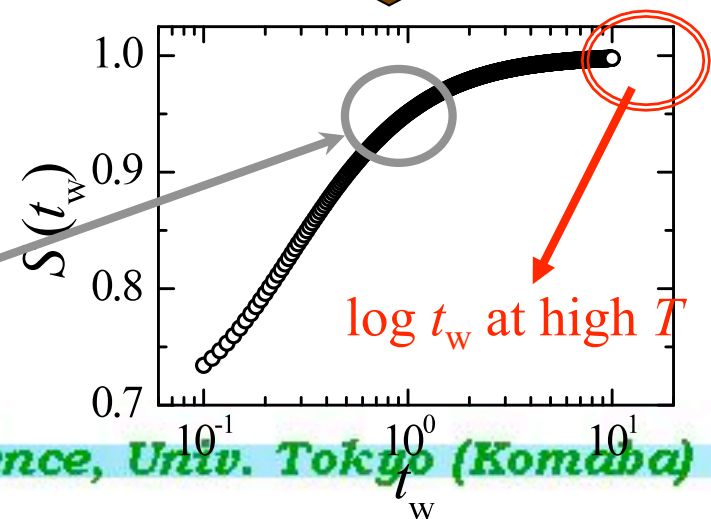


In TAFF region, spatial magnetic profile obeys diffusion equation of motion

$$\frac{\partial B}{\partial t} \approx \frac{c^2}{4\pi} \rho \Delta B \quad \text{Blatter *et al.* RMP (94).}$$

This timescale can be estimated as

$$A_{sample} D^{-1} = A_{sample} \left( \frac{c^2}{4\pi} \rho \right)^{-1} \approx 5 \times 10^{-1} \text{ s}$$

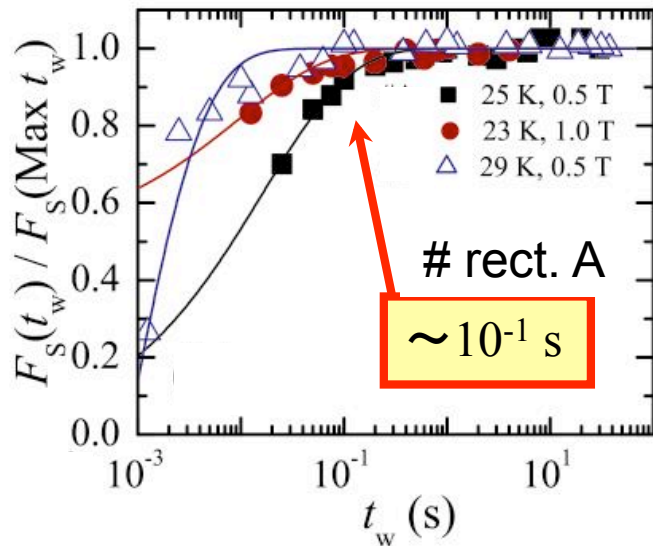


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D. Nakamura *et al.*: arXiv 0906.3086.

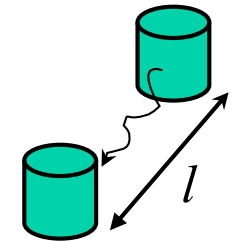
# Rapid relaxation at low $T$ with characteristic timescale

Low temp. Slow  $t_w^*$  cannot be explained by the motion of a single vortex



## Bundle relaxation

$h$  : thickness,  $\eta$ : viscous coefficient  
 $\xi$ : coherence length



$$E_{diffusion} \approx E_{depinning}$$

$$h \cdot \eta \frac{l}{t_w^*} \cdot l = j_c \Phi_0 \cdot h \cdot \xi^* \rightarrow l \approx \sqrt{\frac{j_c \Phi_0 t_w^* \xi^*}{\eta}}$$

energy loss by the viscous motion

depinning energy

$l \approx 12 \mu\text{m}$

Length scale of coherent relaxation (cf. vortex-vortex pacing :  $\sim 50\text{nm}$ )

$$\left. \begin{aligned} t_w^* &\approx 0.1 \text{ s} \\ \eta &\approx 5 \times 10^{-8} \text{ Ns/m}^2 \\ j_c \Phi_0 &\approx 3.62 \times 10^{-8} \text{ N/m} \\ \xi^* &\approx 20 \text{ \AA} \end{aligned} \right\} \rightarrow$$

Dynamic coherence length :  $30\mu\text{m}$

(from noise study in Bi-2212)

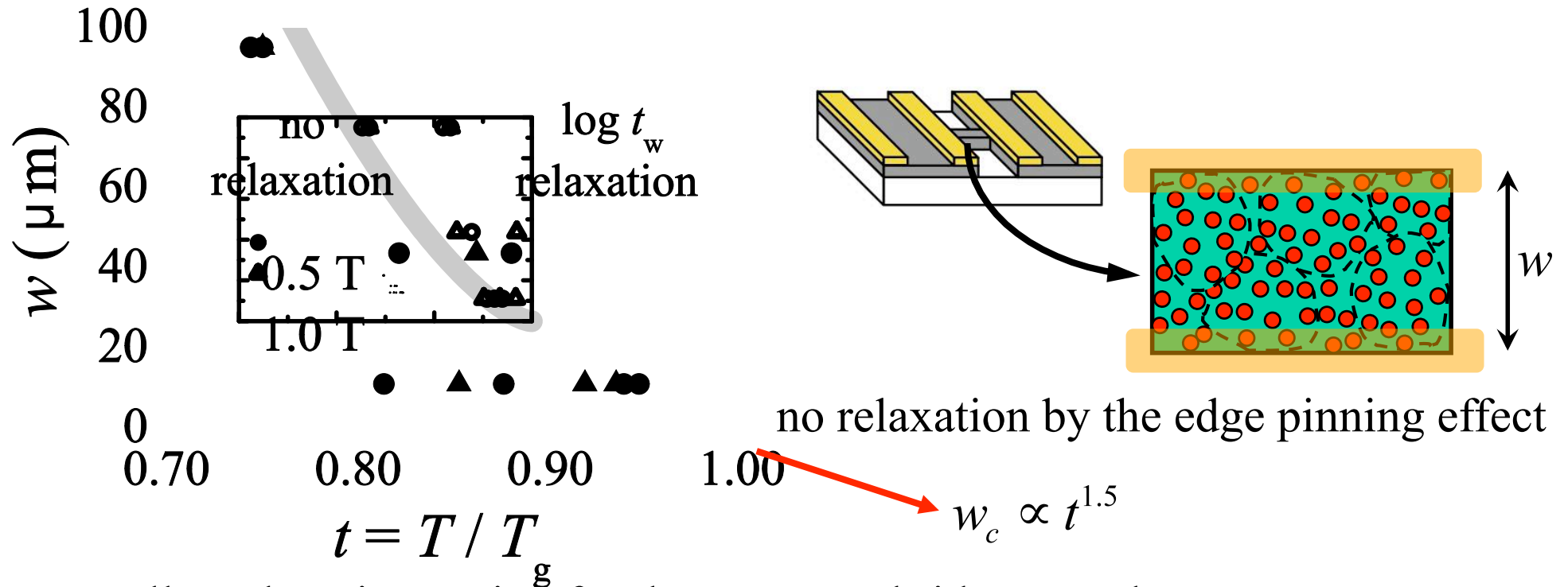
A. Maeda *et al.*, Phys. Rev. B **65**, 054506 (2002).

Bundle relaxation picture confirmed



## Size effect of the relaxation

### crossover line of the relaxation



smaller relaxation region for the narrower bridge sample

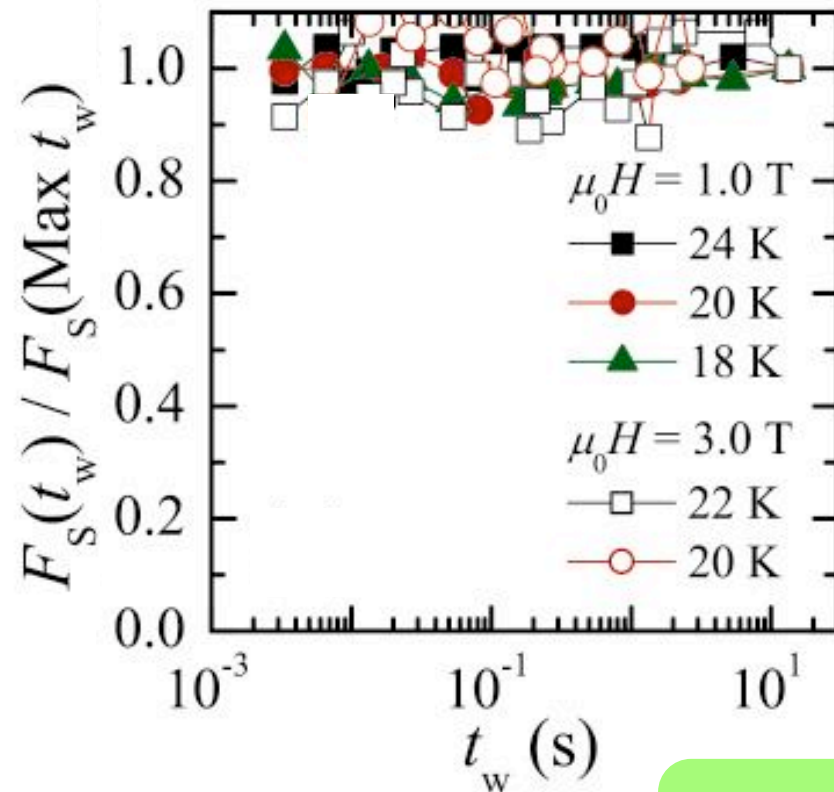
**Interplay between the size of coherent bundle and the system size  
crucially affects the validity of Amontons-Coulomb's law**





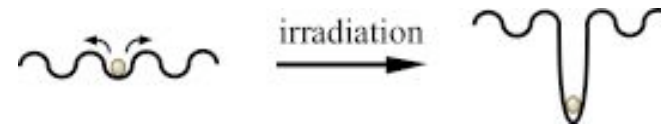
## Bridge sample with columnar defects

	$x$	$T_c$ (K)	$B_\phi$ (T)
# Columnar	0.15	35.11	2.0



**Dramatic changing for the relaxation**

Sample with columnar defects  $\rightarrow$   
no relaxation at all



Strong pinning force also leads to  
the **Amontons-Coulomb's** type friction



## Summary of $F_s(t_w)$ experiments

The size of vortex bundle

	high $T$	low $T$	
Rectangular film	$\log t_w$	characteristic timescale	→ <u>size effect</u>
Bridge film	$\log t_w$	no relaxation	
Bridge film with columnar defects	no relaxation	no relaxation	→ <u>columnar pin effect</u>

### The origin of the relaxation phenomena

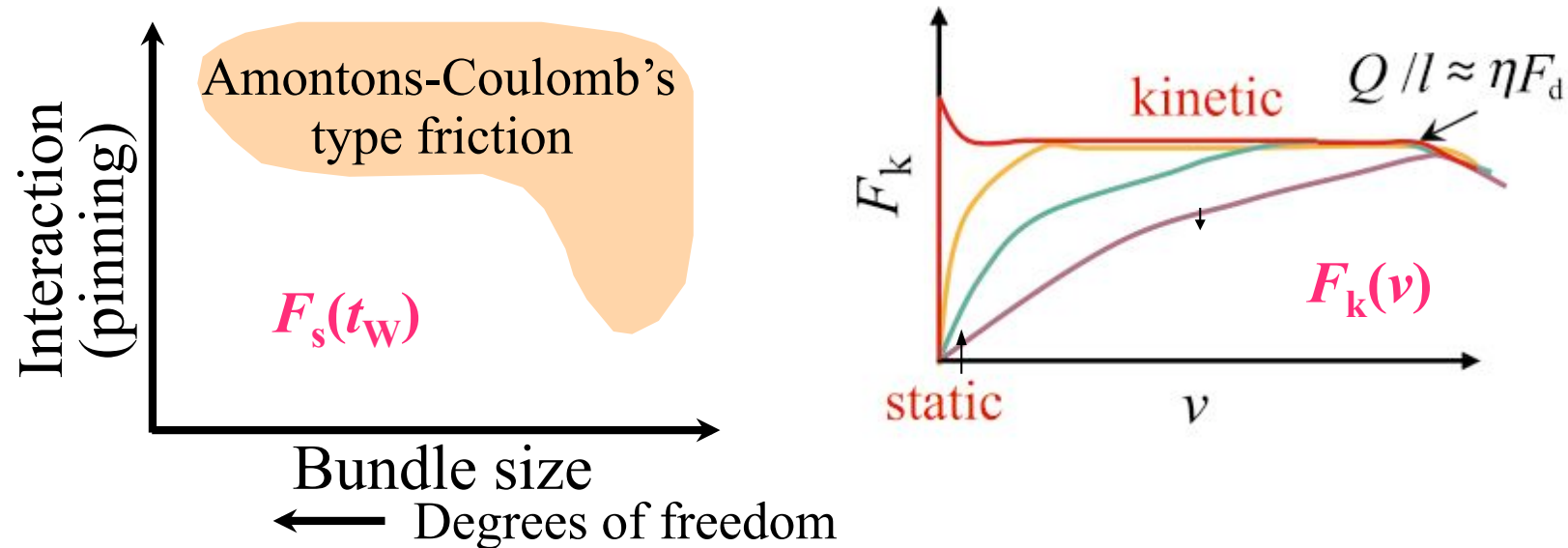
Thermally Assisted Flux Flow of vortex bundle

Bundle size determines the characteristic timescale

no relaxation  $\longleftrightarrow$  very strong pinning center  
(Amontons-Coulomb like)



## Our achievements on this model approach



Key parameters of the validity of Amontons-Coulomb's law

- pinning strength and thermal fluctuation
- size of the coherently moving object and system size

a universal parameter :  $C \equiv \frac{L}{R_c} \exp[-U/k_B T_{\text{eff}}] \quad T_{\text{eff}} \propto T/L^3$

a criteria of the validity of A-C law :  $C \ll C_{cr}$ ,  $C_{cr}$  : critical value



## Friction vs ac dynamics

Friction at the interface  $\longleftrightarrow$  Dissipation by elementary excitation at the interface

Ac dynamics at the interface  
vs microscopic understanding of friction

1) Importance of lattice dynamics

Phonons

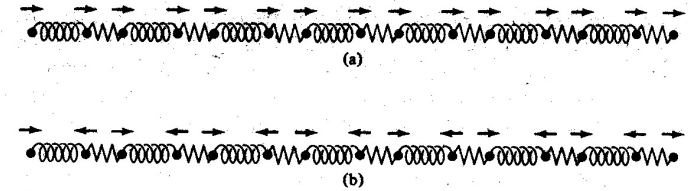
Phonons in disordered systems

Corresponding excitations in CDWs and vortices in SC

2) data at  $\mu$ -waves for vortices



# Normal modes by periodic array of atoms: Phonon



$\omega = \omega(k)$  Dispersion relation

$\omega$  Angular frequency

$k$  Wave number (well defined)

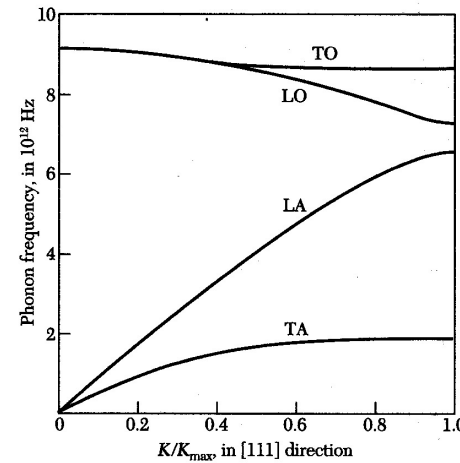
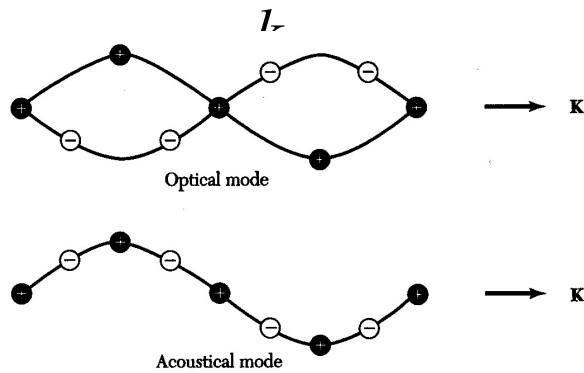
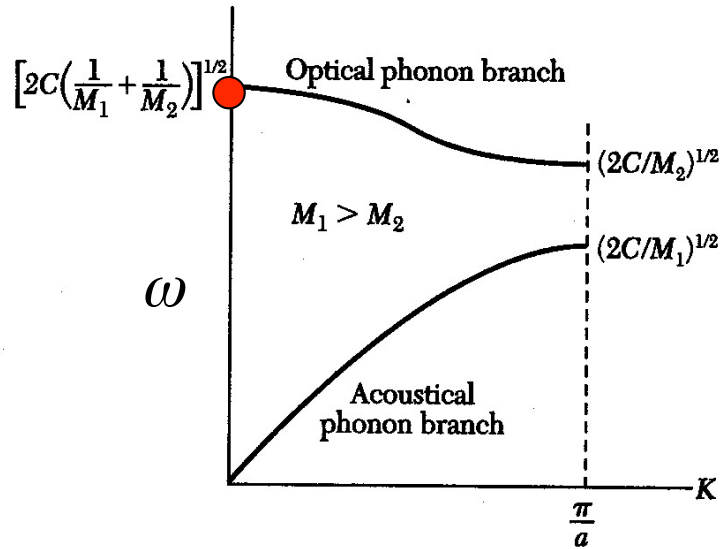


Figure 8a Phonon dispersion relations in the [111] direction in germanium at 80 K. The two TA phonon branches are horizontal at the zone boundary position,  $K_{\max} = (2\pi/a)(\frac{1}{2}\frac{1}{2}\frac{1}{2})$ . The LO and TO branches coincide at  $K = 0$ ; this also is a consequence of the crystal symmetry of Ge. The results were obtained with neutron inelastic scattering by G. Nilsson and G. Nelin.

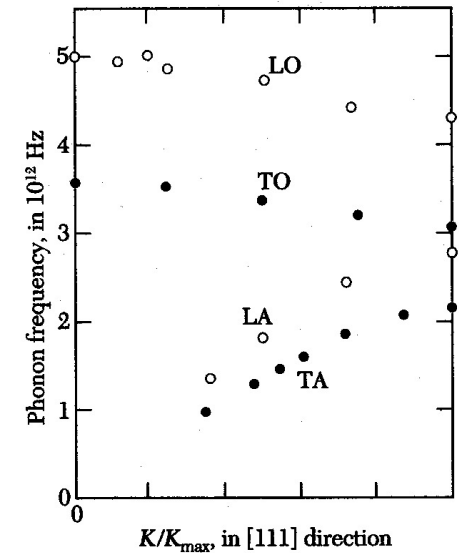


Figure 8b Dispersion curves in the [111] direction in KBr at 90 K, after A. D. B. Woods, B. N. Brockhouse, R. A. Cowley, and W. Cochran. The extrapolation to  $K = 0$  of the TO, LO branches are called  $\omega_T, \omega_L$ .

(C. Kittel : "Introduction to Solid State Physics")



## Optical response by a bound mode: Lorentz oscillator

$$m \ddot{x} + m\Gamma \dot{x} + m\omega_0^2 x = -eEe^{i\omega t}$$

$$\tilde{\varepsilon} = 1 + \frac{Ne^2}{\varepsilon_0 m} \frac{1}{(\omega_0^2 - \omega^2) - i\Gamma\omega}$$

$$\tilde{\varepsilon} \equiv \varepsilon_1 + i\varepsilon_2$$

$$\varepsilon_1 = 1 + \frac{Ne^2}{\varepsilon_0 m} \frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (\Gamma\omega)^2}$$

$$\varepsilon_2 = \frac{Ne^2}{\varepsilon_0 m} \frac{\Gamma\omega}{(\omega_0^2 - \omega^2)^2 + (\Gamma\omega)^2}$$

$$\sigma_1(\omega) = \omega\varepsilon_2(\omega) \propto \omega^2 \quad \text{for small } \omega$$

(F. Wooten : "Optical Properties of Solids")

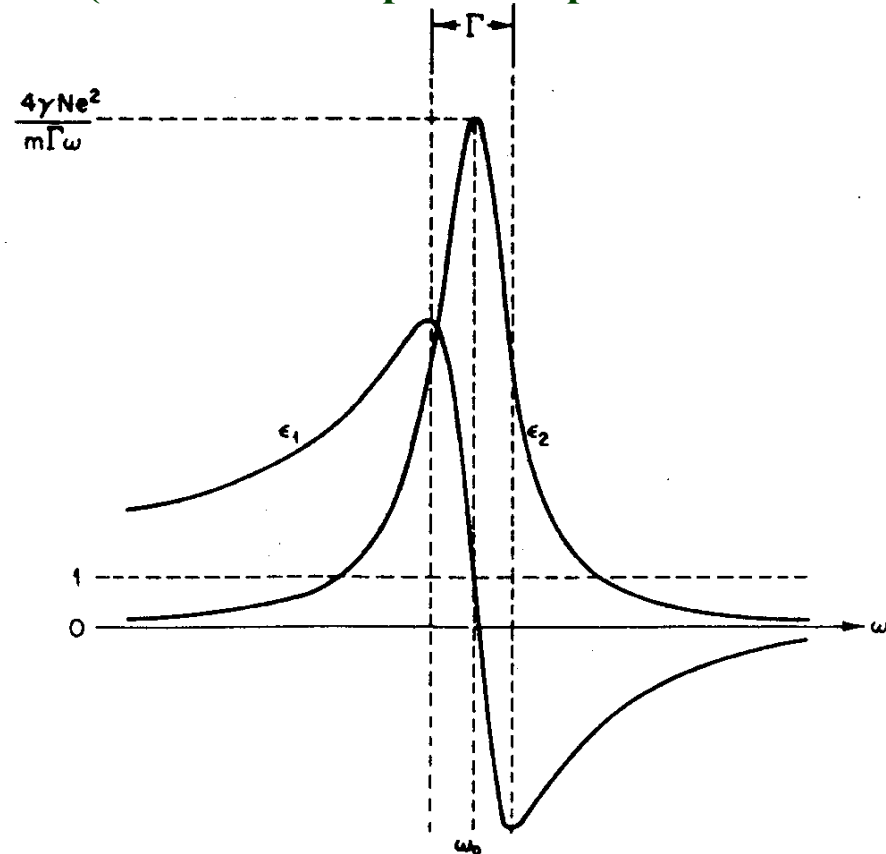


Fig. 3.1 Frequency dependence of  $\varepsilon_1$  and  $\varepsilon_2$ .

**For pinned CDW**

H. Fukuyama and P. A. Lee : Phys. Rev. B17 (1978) 535.

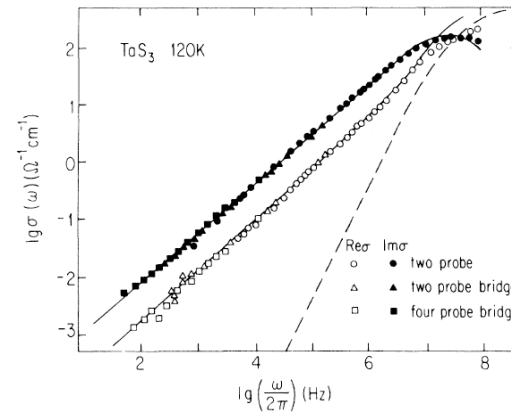
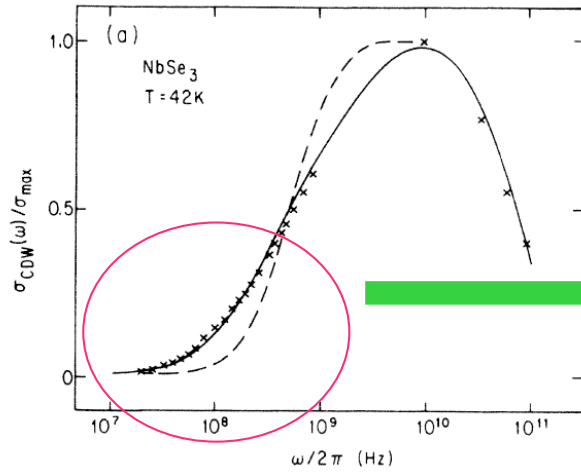
P. A. Lee, T. M. Rice and P. W. Anderson: SSC 14 (1974) 703.



Maeda Lab, Dep. Basic Science, Univ. Tokyo (Komaba)

# Experiments in CDWs

(D. Reagor *et al.*: Phys. Rev. B34 (1986) 2212.) (W. Wu *et al.*: Phys. Rev. Lett. 52 (1984) 2382.)



$$\sigma_1(\omega) \propto \omega^\alpha$$

$$\alpha \cong 0.8$$

$$\epsilon_2(\omega) \propto \omega^{-(1-\alpha)}$$

divergent

FIG. 1.  $\text{Re}\sigma(\omega)$  and  $\text{Im}\sigma(\omega)$  vs frequency at  $T = 120$  K. The dc conductivity has been subtracted from  $\text{Re}\sigma$ . The full lines represent the Cole-Cole-like expression, Eq. (7), with  $\alpha = 0.87$  and  $\tau^{-1}/2\pi = 25$  MHz. The dashed line gives the low-frequency limit of the harmonic oscillator response,  $\text{Re}\sigma \propto \omega^2$ .

(R. J. Cava *et al.*: Phys. Rev. B30 (1984) 3228.)

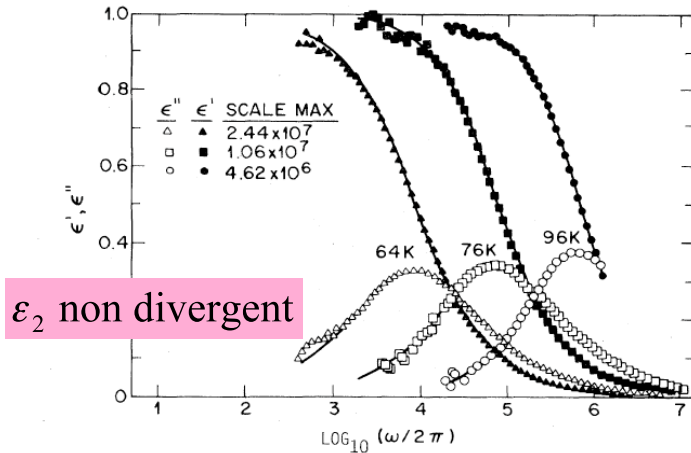


FIG. 6. Frequency dependence (log scale) of the real and imaginary parts of the dielectric constants for the pinned CDW in  $\text{K}_{0.3}\text{MoO}_3$  for three representative temperatures. Solid lines are from the fits of Eq. (2) to the data.

(R. J. Cava *et al.*: Phys. Rev. B31 (1985) 8325.)

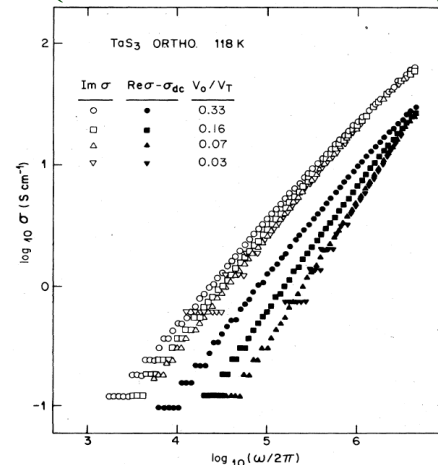


FIG. 1. Low-frequency conductivity of orthorhombic  $\text{TaS}_3$  at 118 K as a function of ac signal level (rms). Open symbols imaginary  $\sigma$ ; closed symbols real  $\sigma - \sigma_{dc}$ .

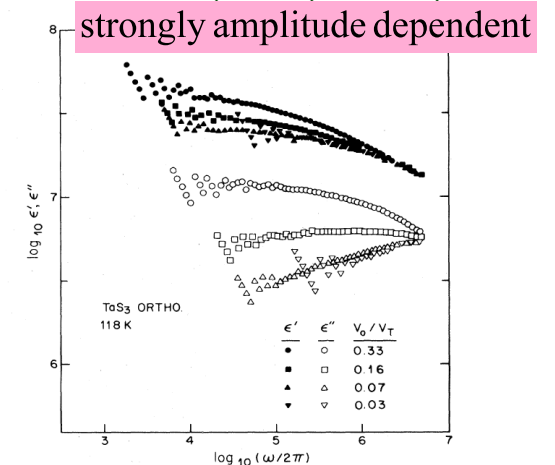


FIG. 2. Low-frequency dielectric response of orthorhombic  $\text{TaS}_3$  at 118 K as a function of ac signal level (rms). Open symbols:  $\epsilon''(\omega)$ , closed symbols:  $\epsilon'(\omega)$ .

(also J. P. Stokes *et al.*: Phys. Rev. B32 (1985) 6939.)



# Mean-field model of vortex motion

P. Martinoli *et al.* Physica B 165&166, 1163 (1990),  
 M. W. Coffey and J. R. Clem: PRL 67 (1991) 386, PRB 45 (1992) 9872.

$$\eta \dot{\mathbf{u}}(\mathbf{r}, t) + \kappa_p \mathbf{u}(\mathbf{r}, t) = \mathbf{j}(\mathbf{r}, t) \times \Phi_0 \hat{\mathbf{a}} + f_T(t)$$

$$\mathbf{j} = \mathbf{j}_n + \mathbf{j}_s$$

$$\mathbf{j}_n = \sigma_{nf} \mathbf{E}$$

$$\mathbf{j}_s = -\frac{1}{\mu_0 \lambda_l^2} \mathbf{A} + C_o \nabla \varphi$$

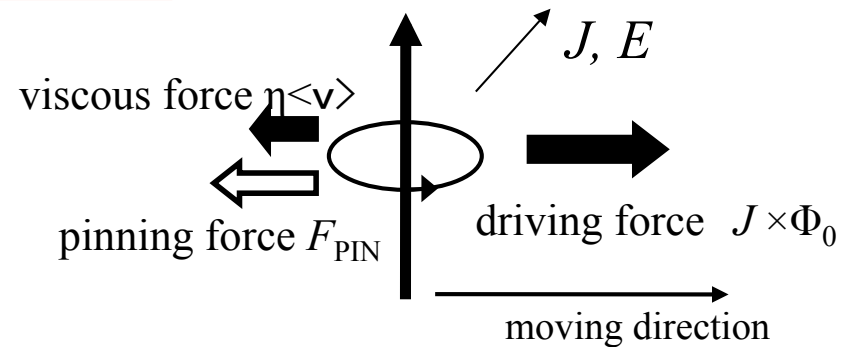
$$C_o = \frac{\Phi_0}{2\pi\mu_0 \lambda^2}$$

remarks

- (a) Magnus force, vortex-vortex interaction are not considered.
- (b) Vortex **mass** is considered to be **zero**.
- (c) Condensate wave function is considered to be *s*-wave.
- (d) From microscopic point of view, the two-fluid picture is not a good approximation.

(Eschrig-Rainer-Sauls)

massless  
overdamped

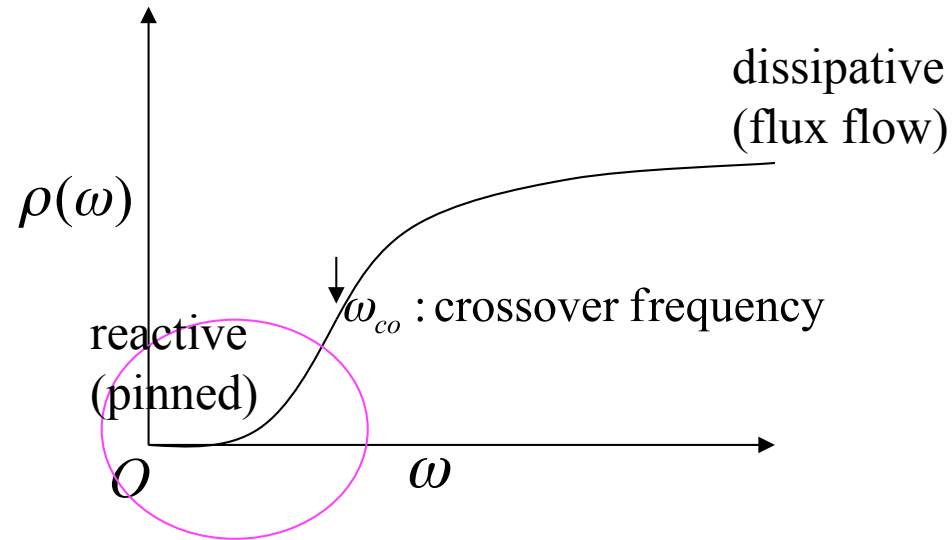




## Ac resistivity by the vortex motion

$$E(\omega) = \rho(\omega) j(\omega)$$

$\rho(\omega)$  : resistivity



$\omega^2$  ?

$\omega^\alpha$  ? ( $\epsilon_2$  divergent)

strongly nonlinear (amplitude dependent)?



## Our expectation

Non AC like

$\rho(\omega)$

Weaker pinning

Divergent  $\varepsilon(\omega)$

Smaller bundle size

Remarkable nonlinearity

Stronger pinning

Lorentz-oscillator like  $\varepsilon(\omega)$

Larger bundle size

no nonlinearity

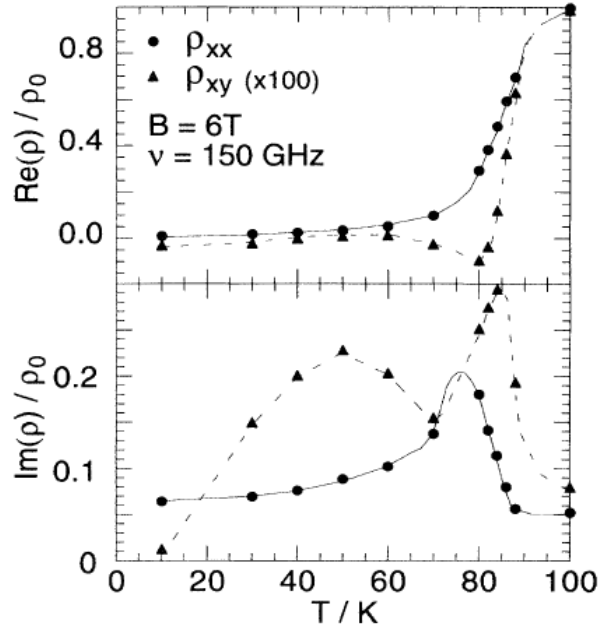
AC like



# Experiments in vortices in superconductors (THz)

overdamped

B. Parks *et al.* Phys. Rev. Lett. 74 (1995) 3265.



YBCO

FIG. 1. Real and imaginary parts of  $\rho_{xx}$  and  $\rho_{xy}$ . Despite scaling by  $10^2$ , the sign changes in  $\rho_{xy}$  from negative to positive near 70 K and back to negative near 20 K are barely discernable. All resistivities are normalized to  $\rho_0$ , the dc resistivity at 100 K. The data shown correspond to the 40 nm film on  $\text{LaAlO}_3$ . The lines are guides to the eye.

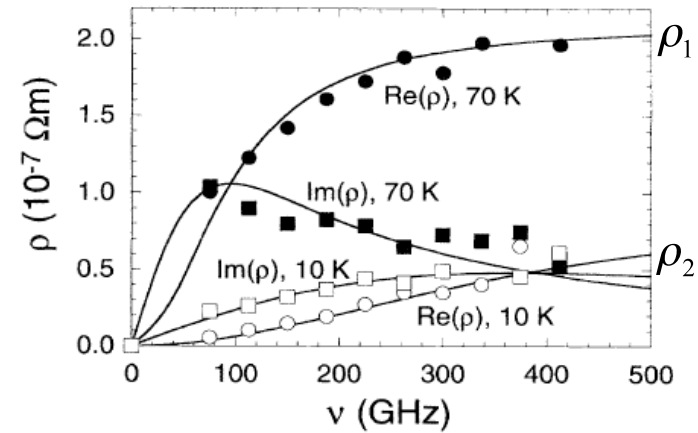


FIG. 2. The vortex contribution to the resistivity plotted vs frequency at 10 and 70 K for  $B = 6$  T. The lines are fits to the data using Eq. (3a). The data shown were taken on the 70 nm film on  $\text{CeO}_2/\text{Al}_2\text{O}_3$ .

$$X_v = \omega \phi_0 B / \kappa$$

$$\Gamma \equiv \kappa / \eta$$

$$\rho_v^{xx} = iX_v \frac{(1 + i\omega/\Gamma)}{(1 + i\omega/\Gamma)^2 - (\alpha\omega/\kappa)^2},$$

$$\rho_v^{xy} = X_v \frac{\alpha\omega/\kappa}{(1 + i\omega/\Gamma)^2 - (\alpha\omega/\kappa)^2},$$

asic

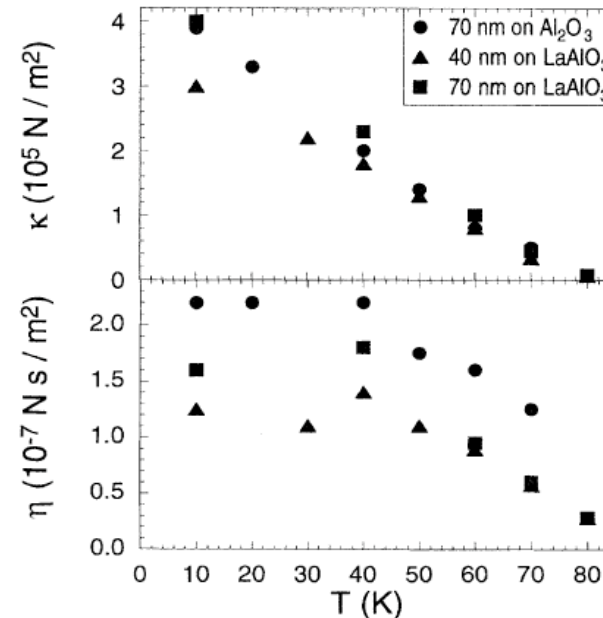


FIG. 3. The vortex pinning and damping parameters  $\kappa$  and  $\eta$  plotted as functions of temperature for all three samples.

# Experiments in vortices in superconductors ( $\mu$ -wave)

overdamped

## Conventional superconductor

Pb-In, Nb-Ta

(J. C. Gittleman and B. Rosenblum:  
Phys. Rev. Lett. 16 (1968) 734.)

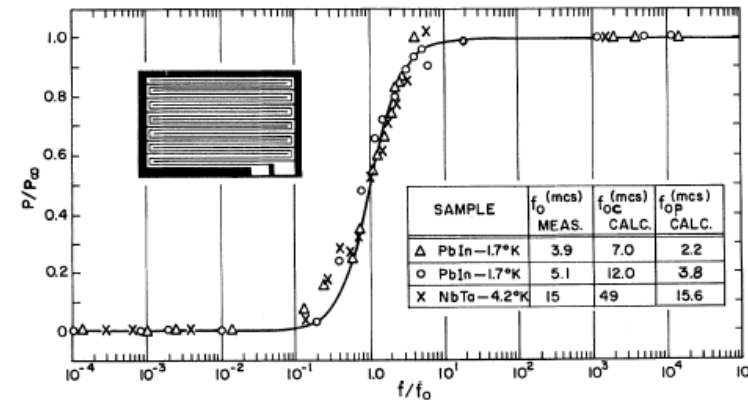
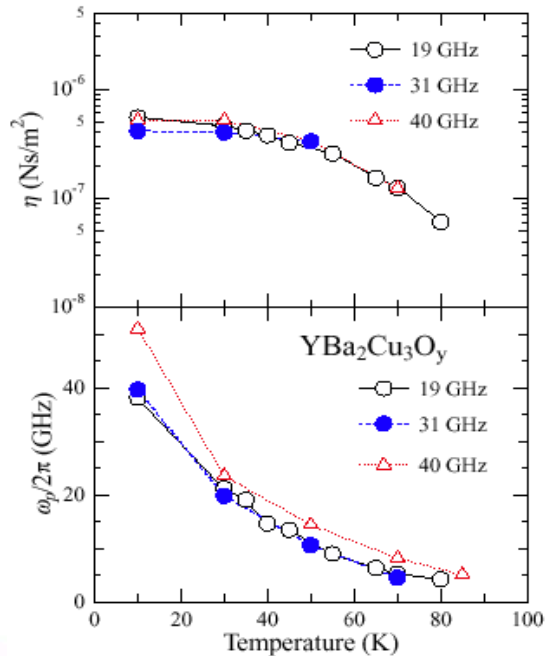


FIG. 1. Power absorption in the mixed state for subcritical currents as a function of frequency. Points at very high frequencies are microwave data taken on other samples of the same materials.

## High- $T_c$ superconductor

(Y. Tsuchiya *et al.*: Phys. Rev. B63 (2001) 184517-1.)

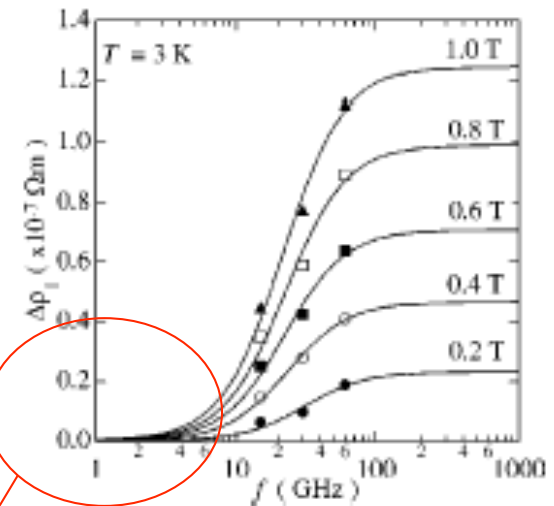
(Y. Matsuda *et al.*: PRB66(2002)014527.)



YBCO

TBCO

Well described  
by the mean-field model  
with **field independent  $\omega_{co}$**



Seems to exhibit **no apparent nonlinearity** at present

Need to be investigated systematically in more detail



# Ac impedance of SC: vortex motion plus superfluid

M. W. Coffey and J. R. Clem, PRL 67, 386 (1991).

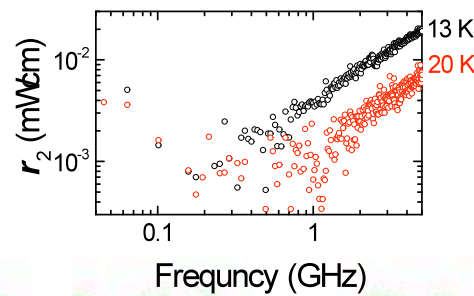
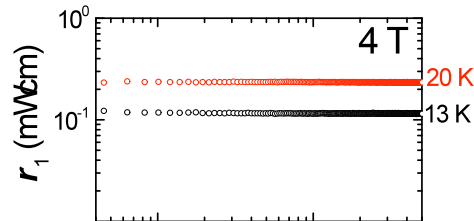
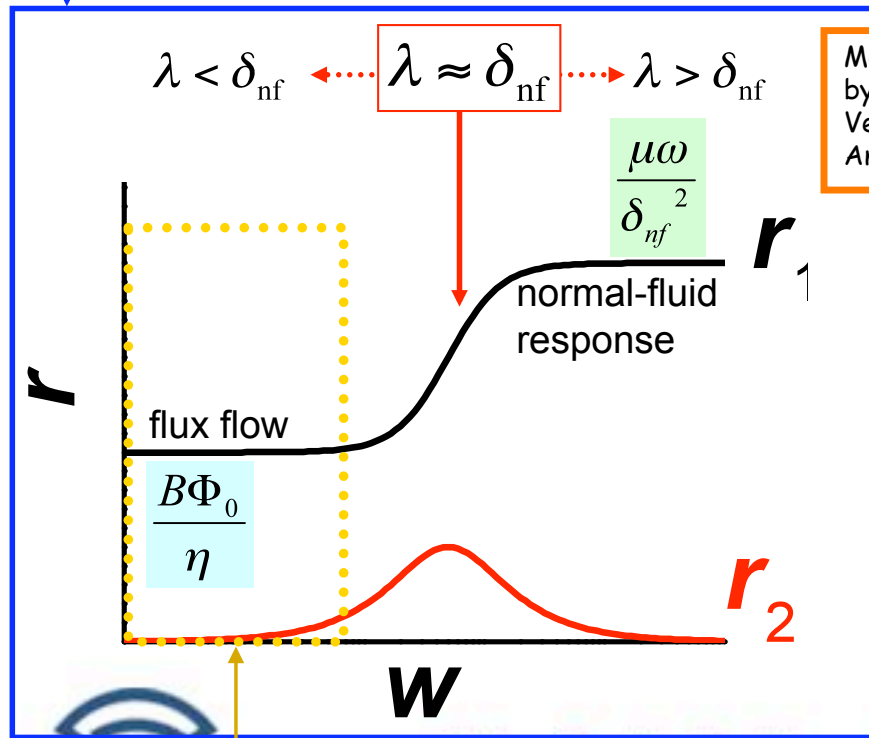
$$\rho(\omega) = \frac{\rho_{\text{vortex}}(\omega) + i\mu_0\omega\lambda^2}{1 + 2i\lambda^2 / \delta_{\text{nf}}(\omega)^2}$$

$\delta_{\text{nf}}$  : normal-fluid skin depth  
 $\lambda$  : penetration depth

$$\rho_{\text{vortex}}(\omega) = \frac{B\Phi_0}{\eta} \left[ \frac{\varepsilon(\omega) + (\omega\tau_p)^2 + i(1 - \varepsilon(\omega))\omega\tau_p}{1 + (\omega\tau_p)^2} \right] \approx \frac{B\Phi_0}{\eta}$$

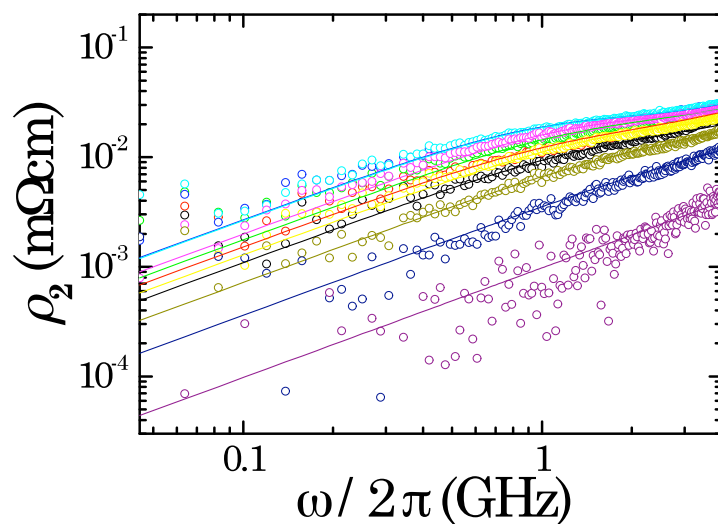
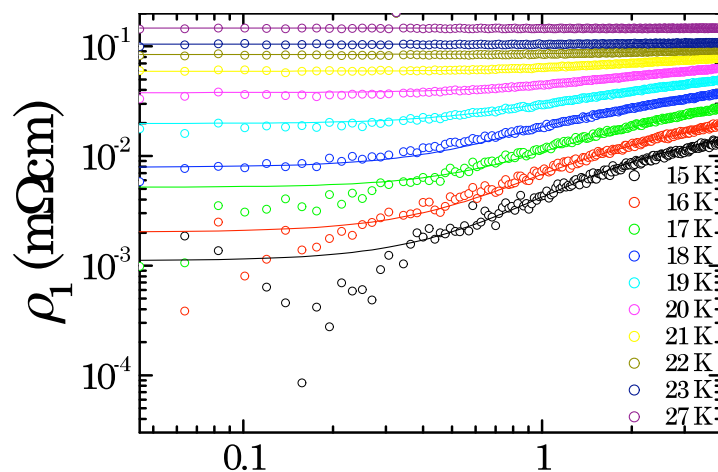
## Parameters

- $\eta$  : viscous drag coeff. } from  $\rho_1$
- $\tau_p^{-1}$  : pinning frequency } from  $\rho_1$
- $\varepsilon$  : flux-creep factor } from  $\rho_1$
- $\lambda$  : penetration depth — from  $\rho_2$

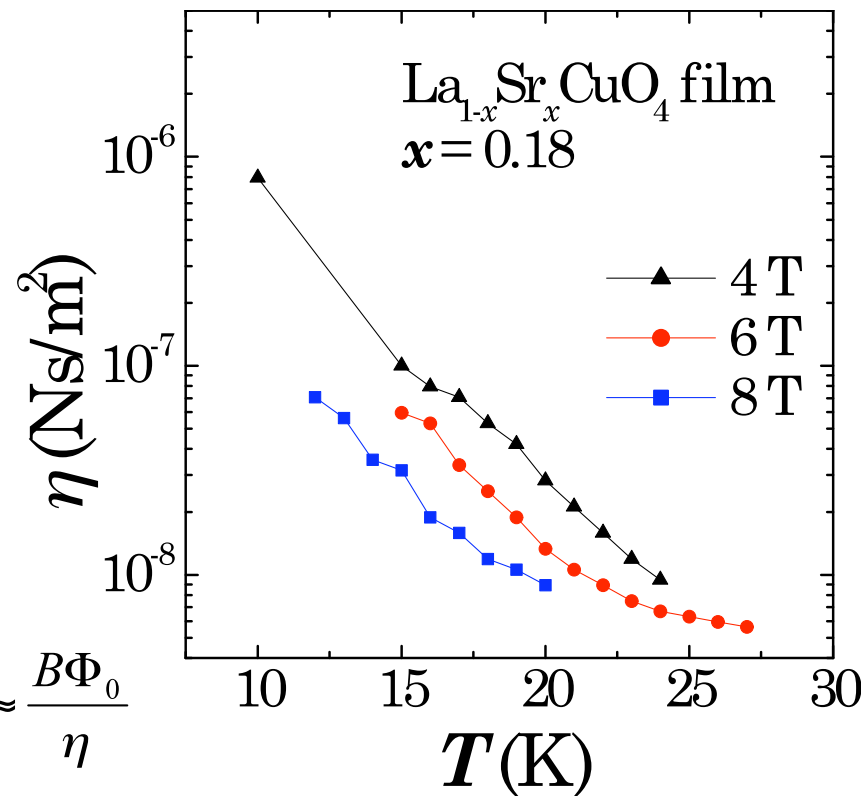


$$\begin{aligned} d\rho_1 / dT &\sim 0 \\ d\rho_2 / dT &> 0 \\ \downarrow \\ \lambda &\ll \delta_{\text{nf}} \end{aligned}$$

# LSCO: viscous drag $\eta$



$$\rho_{vortex} \approx \frac{B\Phi_0}{\eta}$$



Dissipation: similar to other cuprates (YBCO, BSCCO *etc.*)

$$\ell_{core} \approx \xi_{GL} \ll \ell$$


Y. Tsuchiya *et al.*: PRB 63 (2001) 184517.

A. Maeda *et al.*: JPSJ 76 (2007) 094708.

A. Maeda *et al.*: Physica C 460-562 (2007) 1201.

Basic Science, Univ. Tokyo (Komaba)

$\eta$  depends on  $B$ .



$$\rho = \frac{\rho_{vortex} + i\mu_0\omega\lambda^2}{1 + 2i(\lambda/\delta_{nf})}$$

## Conclusion

(1) Discuss dynamics of driven vortices and physics of friction utilizing driven vortices of superconductor

(2) Kinetic friction as a function of velocity:

understood in terms of simple overdamped oscillator model

**Broadened transition of dynamic phase transition (Amontons-Coulomb behavior)**

suggesting the importance of thermal fluctuation for microscopic friction

(3) Static friction as a function of waiting time: relaxation of vortex bundle

**stronger pinning, large bundle size  $\Leftrightarrow$  Amontons-Coulomb like behavior**

(4) Criteria for the validity of Amontons-Coulomb friction

**Presence of a universal parameter :**  $C \equiv \frac{L}{R_c} \exp[-U/k_B T_{\text{eff}}] \quad T_{\text{eff}} \propto T/L^3$

$$C \ll C_{cr}, \quad C_{cr} : \text{critical value}$$

(5) Ac dynamics ( $\mu$ -wave to THz) of vortices

**no remarkable anomaly nor nonlinearity such as CDW system at present**



## Future perspective

- (1) Change pinning, system size more systematically  
introduce stick-slip motion in driven vortices  
change velocity dependence of kinetic friction largely  
Feedback to physics of friction

—Basic step for understanding friction, control of friction—

- (2) Need to investigate  
correspondence between ac dynamics and other friction properties  
—comparative studies of  $F_k(v)$ ,  $F_s(t_w)$  and  $\sigma(\omega)$ —

