cooperative motion in sheared granular matter

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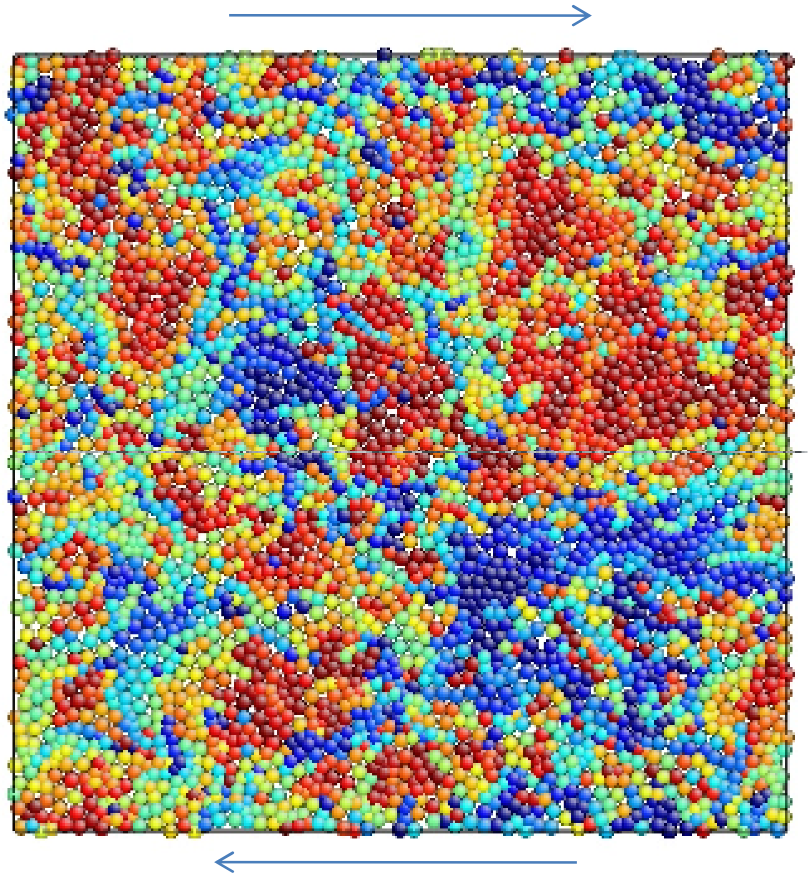
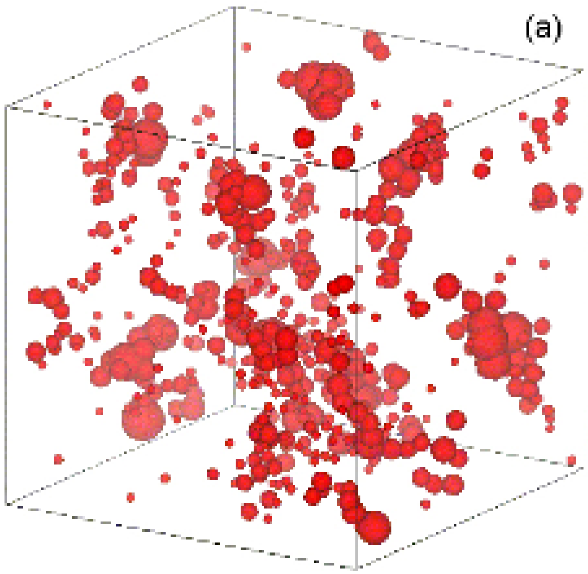
amorphous particulate systems: structure ?

2D granular matter close to jamming



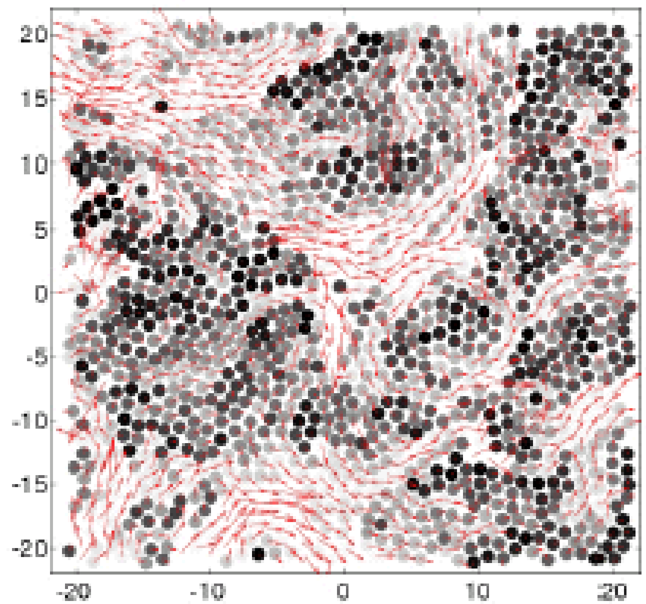
spontaneous formation of long wavelength structure
(detectable using appropriate quantity)

the key: dynamical heterogeneity



supercooled liquid
(Yamamoto & Onuki 1998)

particles with large displacement
during some time window [t, t+dT]

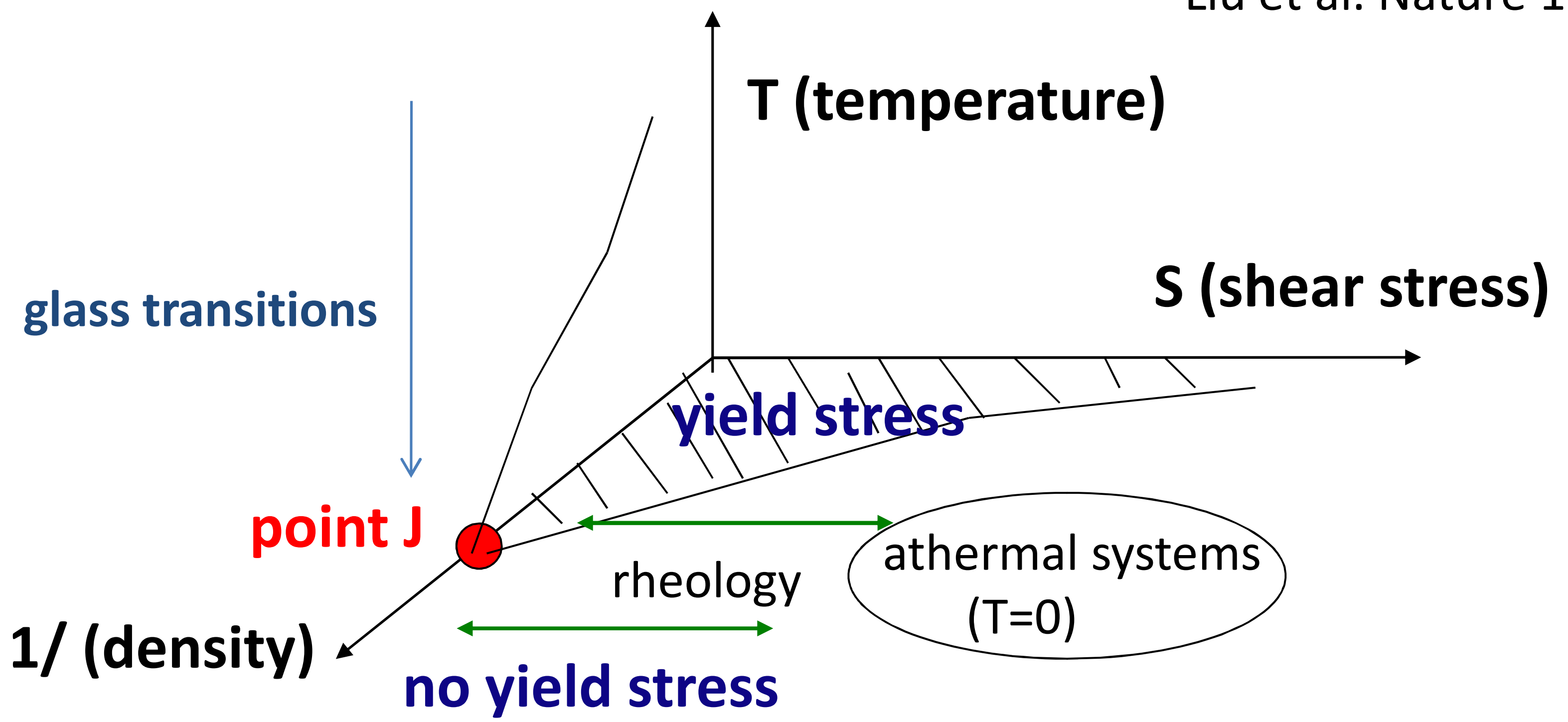


granular matter
(Lechenault at al. EPL 2008)

growing length and time scales → critical point?

“jamming diagram”

Liu et al. Nature 1998



- a critical point located on T=0 plane (athermal systems)
- T=0 critical point dominates the behavior of thermal systems?

NOTE: very controversial
(e.g. Berthier & Witten 2008; Krzakara 2008)

outline of this talk

athermal particulate systems under shear

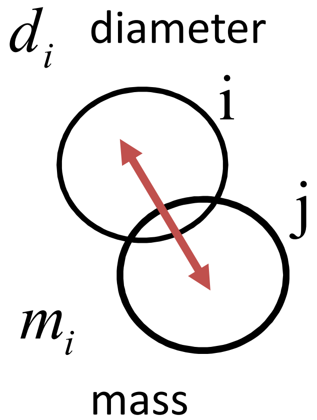
extracting from grain kinetics:

1. intrinsic length scale
   onset of jamming
2. characteristic time scale
   shear thinning

model

an assembly of soft inelastic spheres

(without attractive force & friction)



Force: only normal direction

$$F_{ij} = f(\varepsilon) + \varsigma \vec{n}_{ij} \bullet \vec{v}_{ij}$$

elastic + damping

$$f(\varepsilon) = k\varepsilon$$

relative velocity $\vec{v}_{ij} = \vec{v}_i - \vec{v}_j$

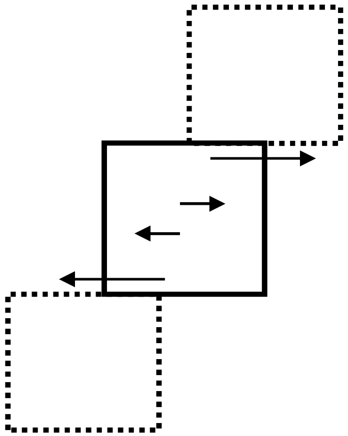
overlap length $\varepsilon = 0.5(d_i + d_j) - r_{ij}$

normal vec. $\vec{n}_{ij} = \dfrac{\vec{r}_i - \vec{r}_j}{\left|\vec{r}_i - \vec{r}_j\right|}$

units m=1 (mass); d = 1 (length); $\sqrt{m/k} = 1$ (time)

geometry, etc

Lees-Edwards Boundary Condition



constant shear rate $\dot{\gamma}$

constant density ϕ

(volume fraction)

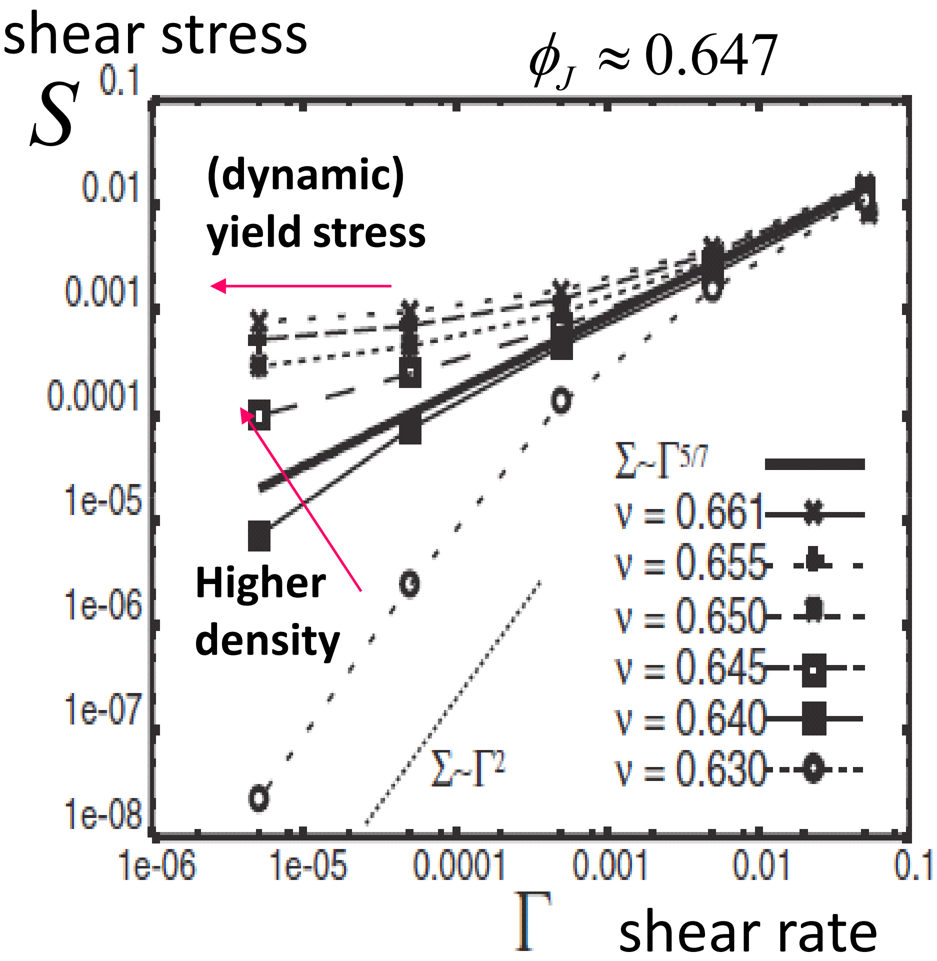
dispersity

bidisperse mixture: d, 0.7d [50:50]

shear stress

$$\sigma_{yz} = V^{-1} \sum_{ij} F_{ij}^{(y)} \bullet r_{ij}^{(z)} \qquad S \equiv \left\langle \sigma_{yz} \right\rangle$$

rheology of athermal particles



(TH, Otsuki, & Sasa, J.Phys.Soc.Jpn. 2007)

1. At lower densities: $\phi < \phi_J$

$S \propto \dot{\gamma}^2$ Bagnold's scaling

(viscosity) ∝ (shear rate)

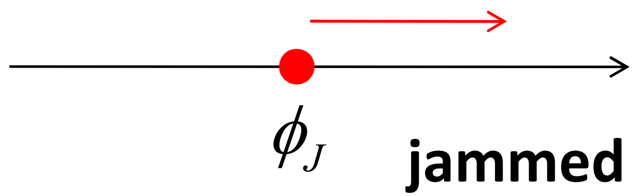
NO yield stress

2. At higher densities: $\phi > \phi_J$

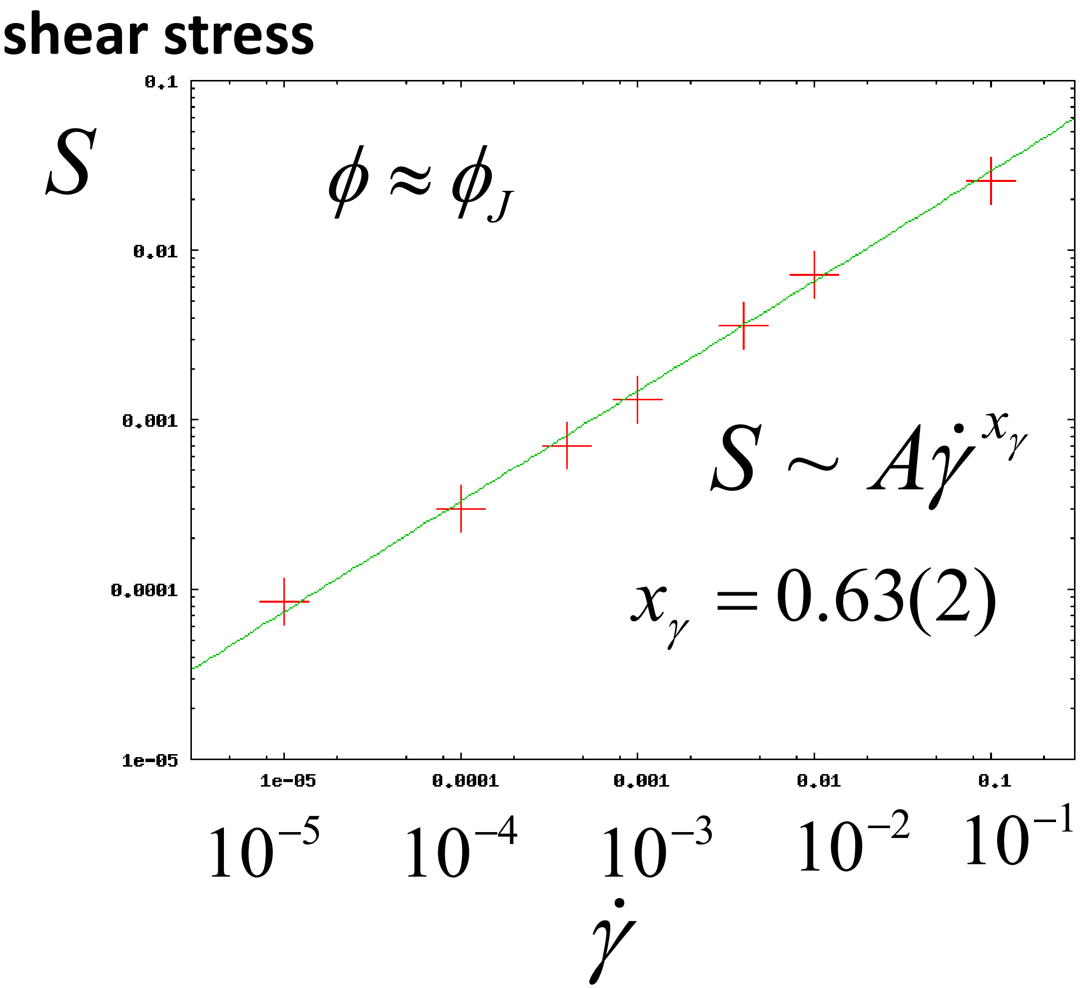
$S \sim S_0 + A\gamma^{HB}$

(Herschel-Bulkley)

dynamic yield stress



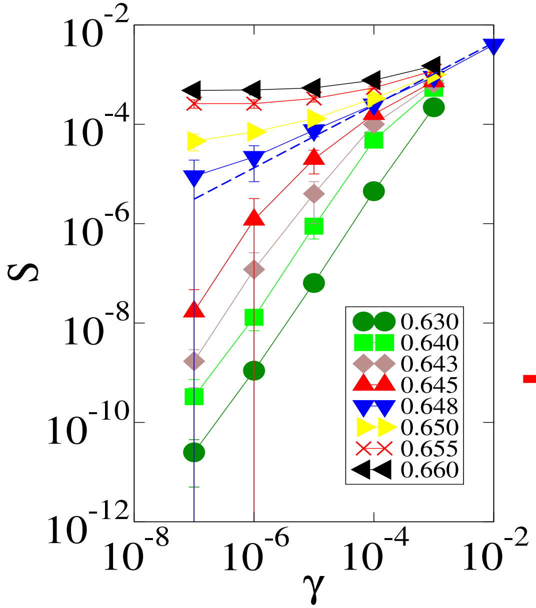
power-law fluid at the onset of jamming

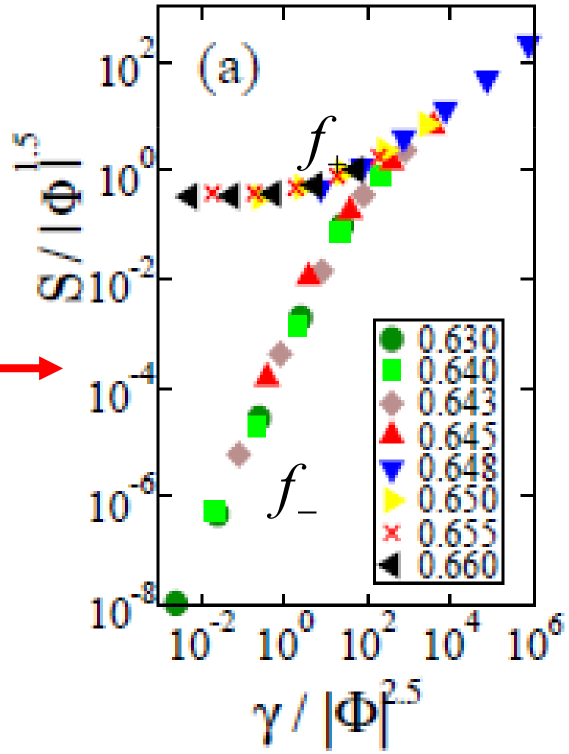


critical scaling of rheology

$$S = |\Phi|^{x_\Phi} f_\pm\left(\frac{\dot{\gamma}}{|\Phi|^{x_\Phi/x_\gamma}}\right)$$

$x_\Phi = 1.5(1)$ $\quad x_\gamma / x_\Phi = 2.5(2)$ $\quad \Phi \equiv \phi_J - \phi$





$f_-(x) \propto x^2$

→ Bagnold's scaling

$f_+(x) \propto const.$

→ yield stress

TH, J.Phys.Soc.Jpn. 2008;
Otsuki & Hayakawa PRE 2009;
TH, Prog. Theor. Phys. 2010;
(cf. Olsson & Teitel 2007)

critical scaling of rheology

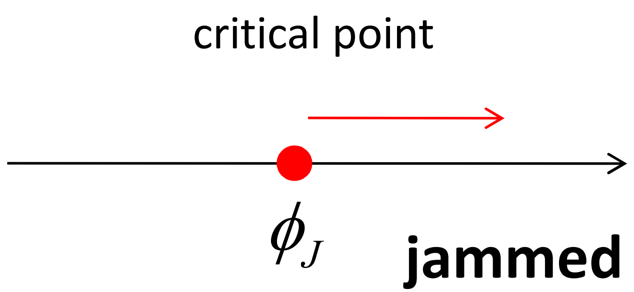
$$S = |\Phi|^{x_\Phi} f_\pm\left(\frac{\dot{\gamma}}{|\Phi|^{x_\Phi / x_\gamma}}\right)$$

$$x_\Phi = 1.5(1) \qquad x_\gamma / x_\Phi = 2.5(2) \qquad \Phi \equiv \phi_J - \phi$$

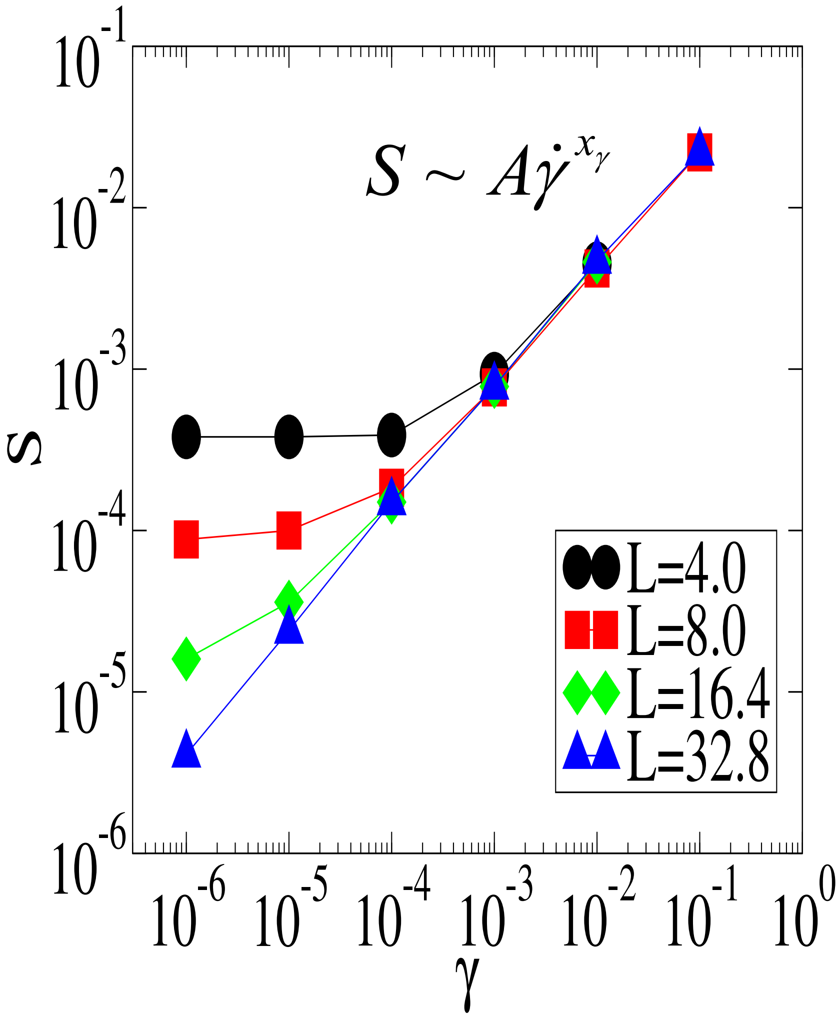
1. yield stress $\qquad S_0 \propto |\Phi|^{x_\Phi} \qquad \because f_+(0) = const.$

2. power-law fluid $\qquad \Phi = 0 \longrightarrow f(x) \sim x^{x_\gamma} \longrightarrow S \propto \dot{\gamma}^{x_\gamma}$

3. shear-thinning $\qquad \dot{\gamma}_{th} \geq |\Phi|^{x_\Phi / x_\gamma}$



finite-size effect in power-law fluid



$\phi = 0.847 \approx \phi_J$

power-law fluid $S \sim A\dot{\gamma}^{x_\gamma}$

system-size dependence
at lower shear rates.

smaller system → higher stress

intrinsic length scale ?

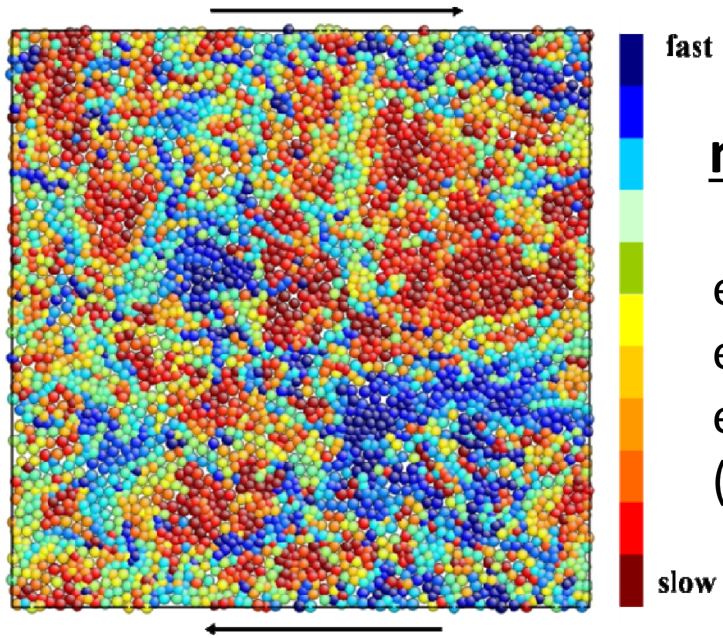
non-affine response in sheared system

$$v_x = \dot{x} \qquad v_y = \dot{y} - \dot{\gamma} z \qquad v_z = \dot{z}$$

- velocity fluctuation wrt. mean flow
- quantifies non-affine motion

non-affine parameter $m_i = \exp\left[-\left(\frac{v_i}{\langle v \rangle} \right)^2 \right]$

nonaffine → 0
affine → 1



<u>non-affine response (blue) is localized</u>

e.g. flow defect (Argon)
e.g. shear transformation zone (Falk & Langer 1998)
e.g. dynamical heterogeneity
(Yamamoto & Onuki 1998, Heuer et al. 1998, etc..)

extracting length scale

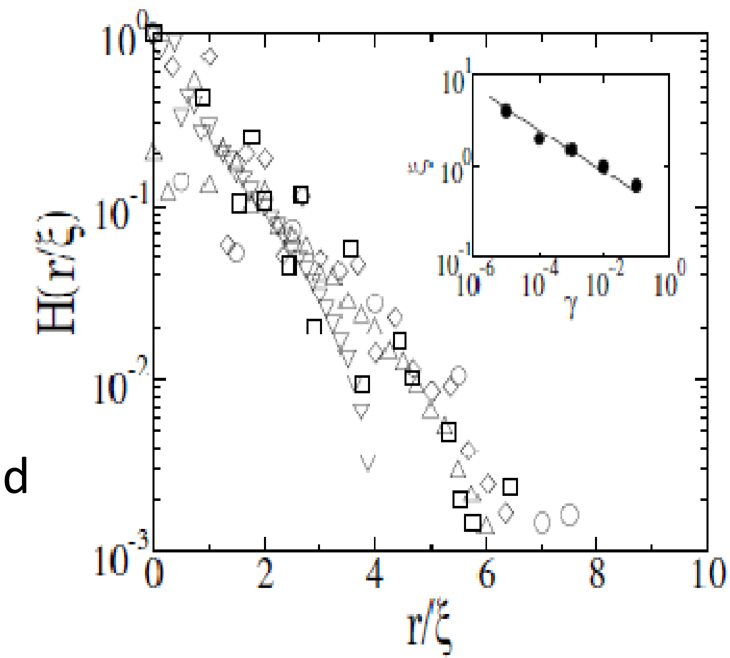
spatial correlation function

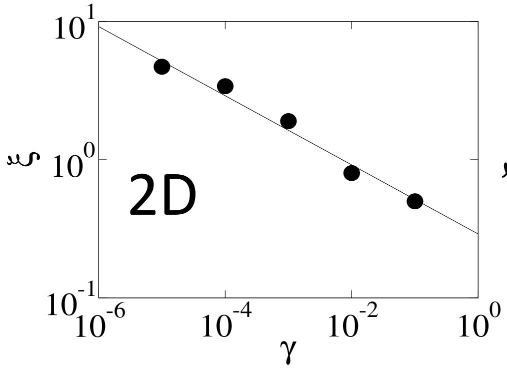
$$h(\vec{r},0) = \frac{1}{N}\sum_{i,j}\delta m_i \delta m_j \delta(\vec{r} - \vec{x}_i + \vec{x}_j)$$

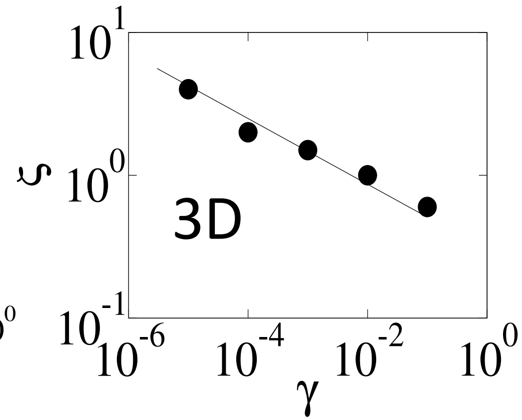
$$m_i = \exp\left[-\left(\frac{v_i}{\langle v \rangle}\right)^2\right]$$

=1 for affine
=0 for non-affine

NOTE: orientation dependence neglected
(cf. Furukawa et al. 2009)







$$\xi \sim \dot{\gamma}^{-y_\gamma}$$

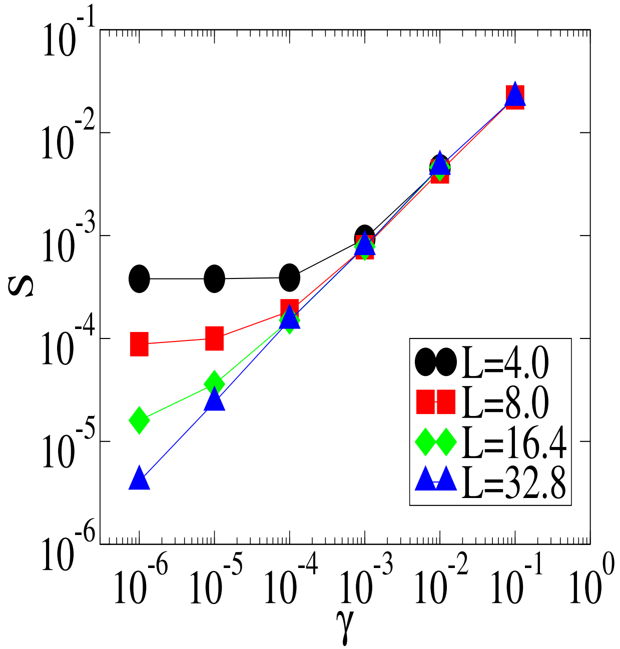
$$y_\gamma = 0.23(2)$$

(TH, arXiv0804.0477)

(also Katsuragi et al., 2010)

the exponent is dimensionality-independent

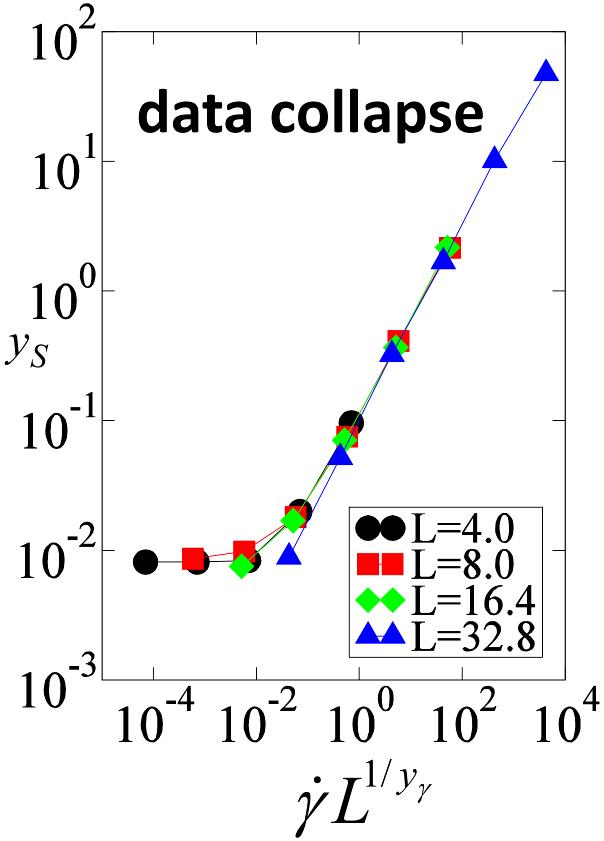
finite-size scaling of rheology



$$S(\dot{\gamma}, L) = L^{-1/y_S} f(\dot{\gamma} L^{1/y_\gamma})$$

$1/y_S = 2.5(3)$

$1/y_\gamma = 4.5(5)$



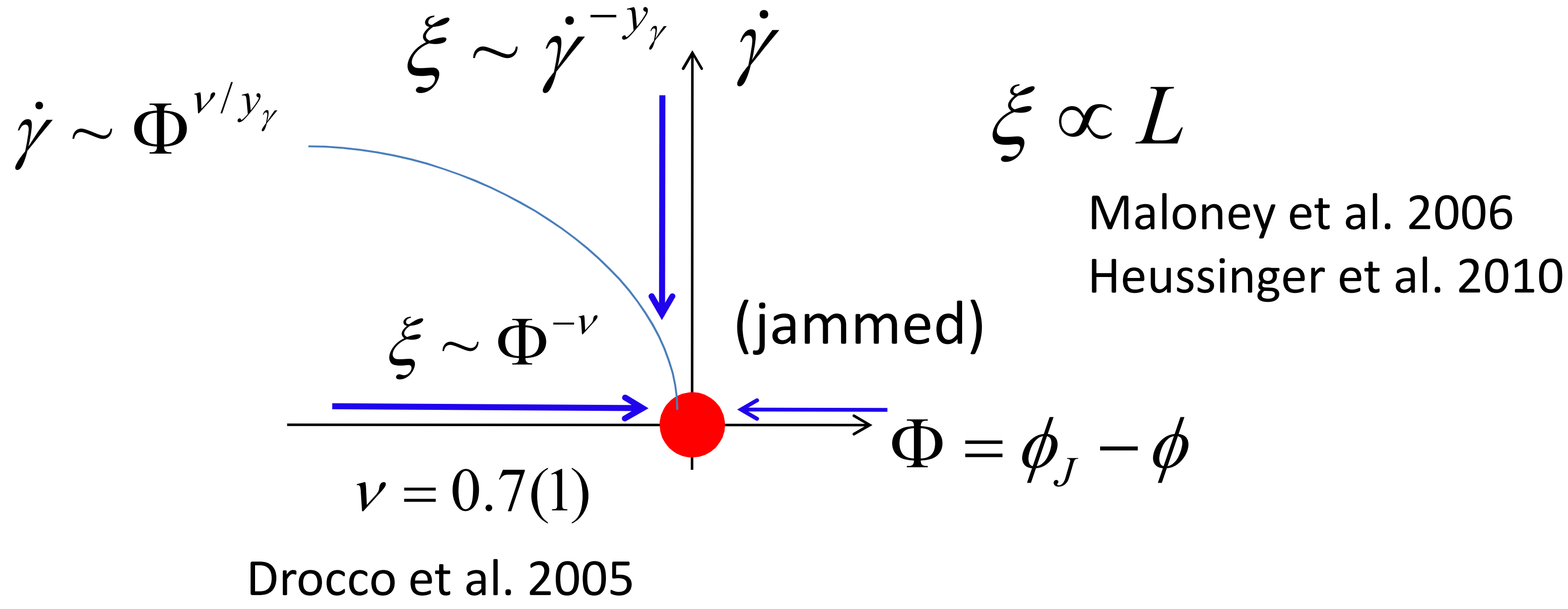
$\dot{\gamma} L^{1/y_\gamma} \sim O(1)$ → yield stress

$$L \sim \dot{\gamma}^{-y_\gamma} \longleftrightarrow L \sim \xi \quad \& \quad \xi \sim \dot{\gamma}^{-y_\gamma}$$

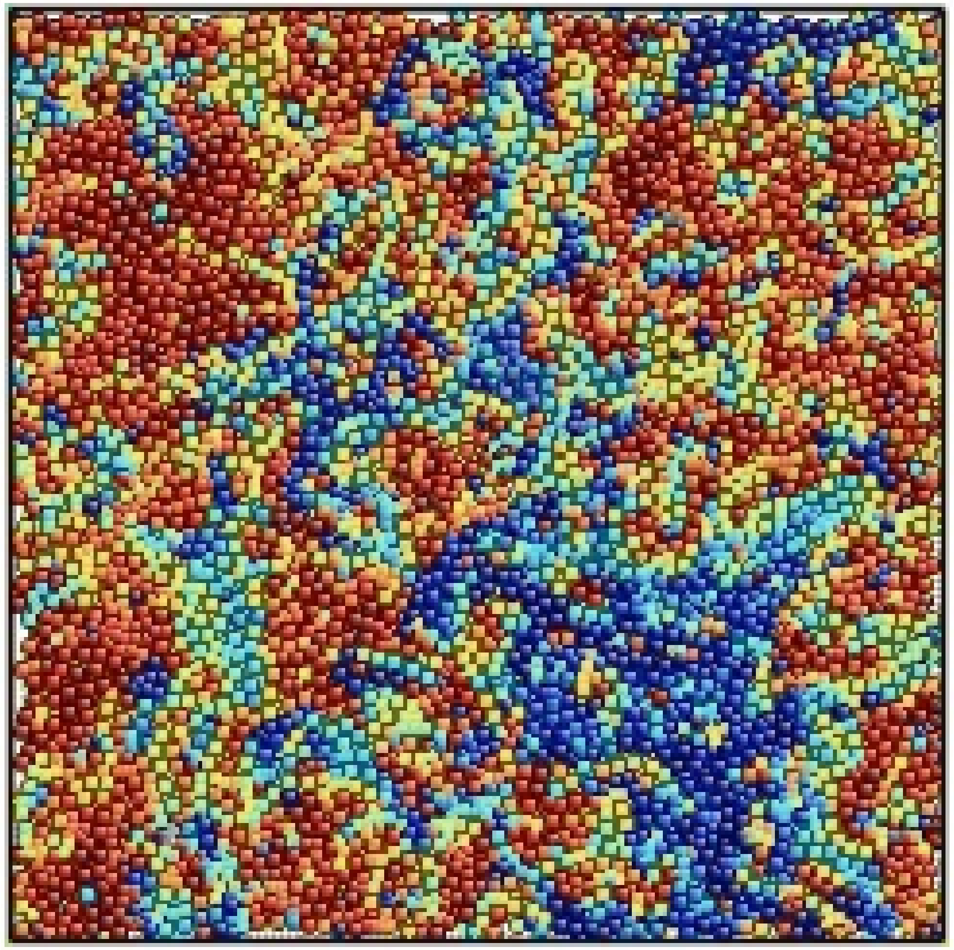
$$y_\gamma \approx 0.22$$

consistent with correlation length

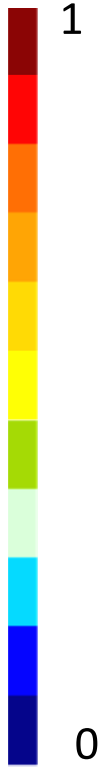
(intrinsic length scale) ~ (system size) → yield stress



extracting time scale from non-affine response



$$m_i(t) = \exp\left[-\left(\frac{v_i(t)}{\langle v \rangle_t} \right)^2 \right]$$

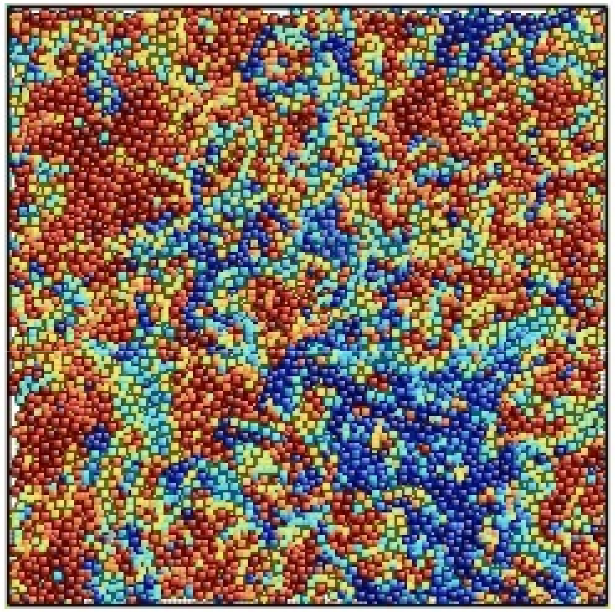


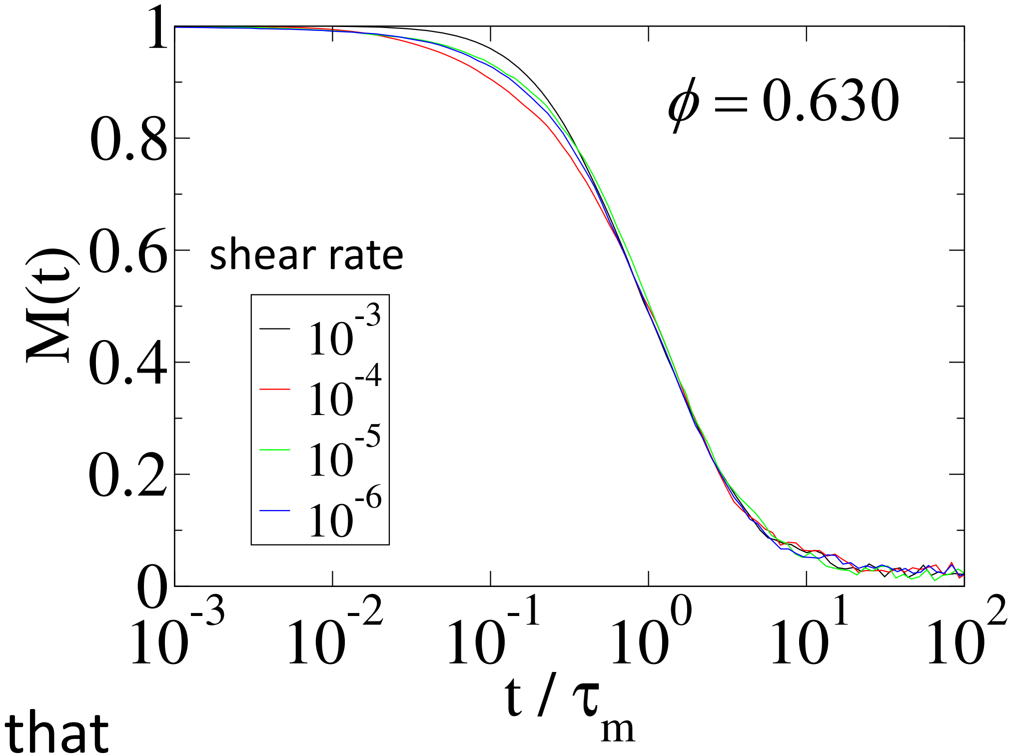
nonaffine → 0
affine → 1

2D $\phi = 0.845 \approx \phi_J$ $\dot{\gamma} = 10^{-6}$ (very close to the critical point)

autocorrelation of non-affine response

$$C_m(t) := \left\langle \delta m_i(t) \bullet \delta m_i(0) \right\rangle / \left\langle \delta m^2 \right\rangle$$



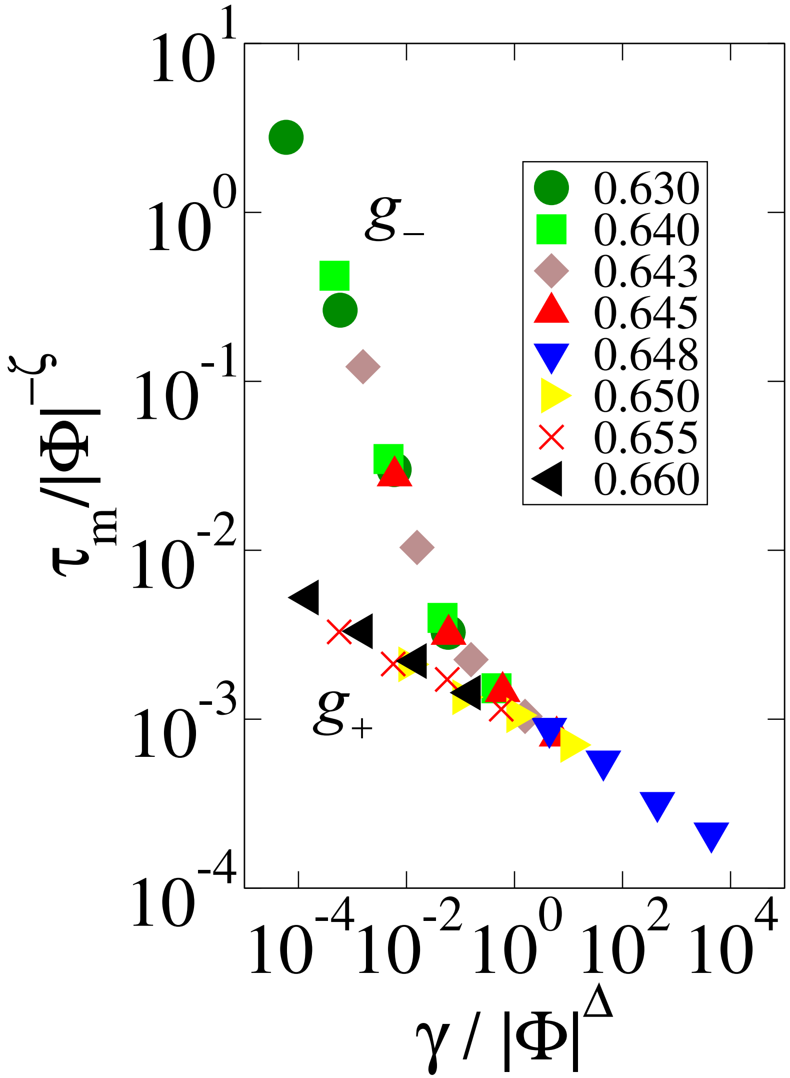


define τ_m in such a way that

$C_m(t/\tau_m)$ collapse to a single curve

→ can obtain $\tau_m(\phi, \dot{\gamma})$

time scale of non-affine response



$$\tau_m = |\Phi|^{-\zeta} g_{\pm}\left(\frac{\dot{\gamma}}{|\Phi|^{\Delta}}\right)$$

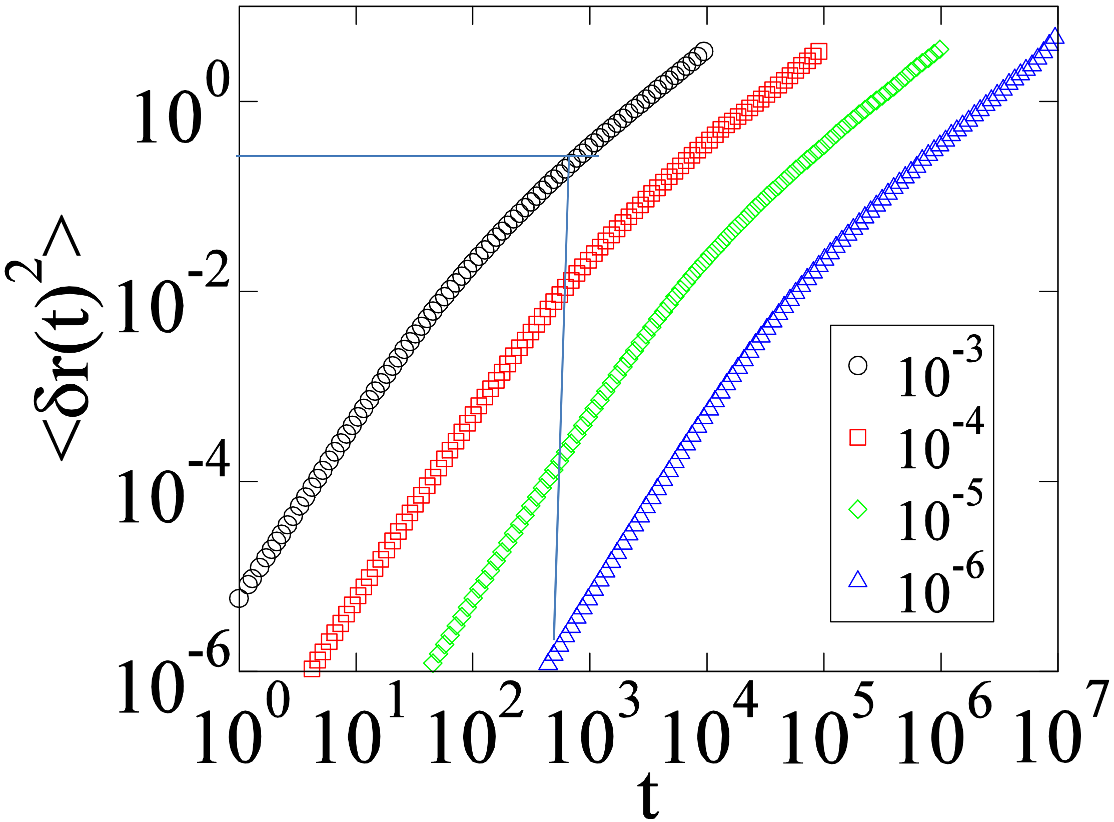
$$\Phi \equiv \phi_J - \phi$$

$$\zeta = 0.6(1) \qquad \Delta = 2.5(1)$$

also obeys scaling law!

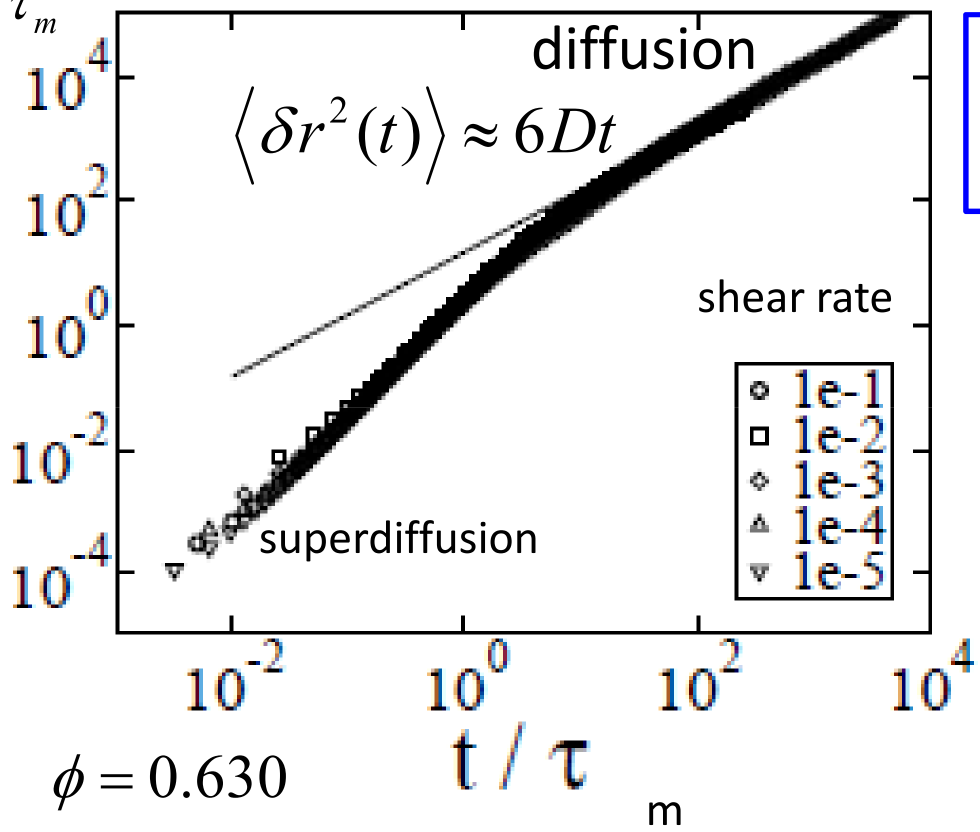
diffusion via non-affine motion

$$\left\langle \delta r^2(t) \right\rangle := \left\langle \left| \int_0^t ds v(s) \right|^2 \right\rangle$$



crossover from superdiffusion to diffusion

scaling of diffusion profile



$$\frac{\langle \delta r^2(t) \rangle}{T\tau_m{}^2}$$

$$\langle \delta r^2(t) \rangle \approx 6Dt$$

$$\phi = 0.630$$

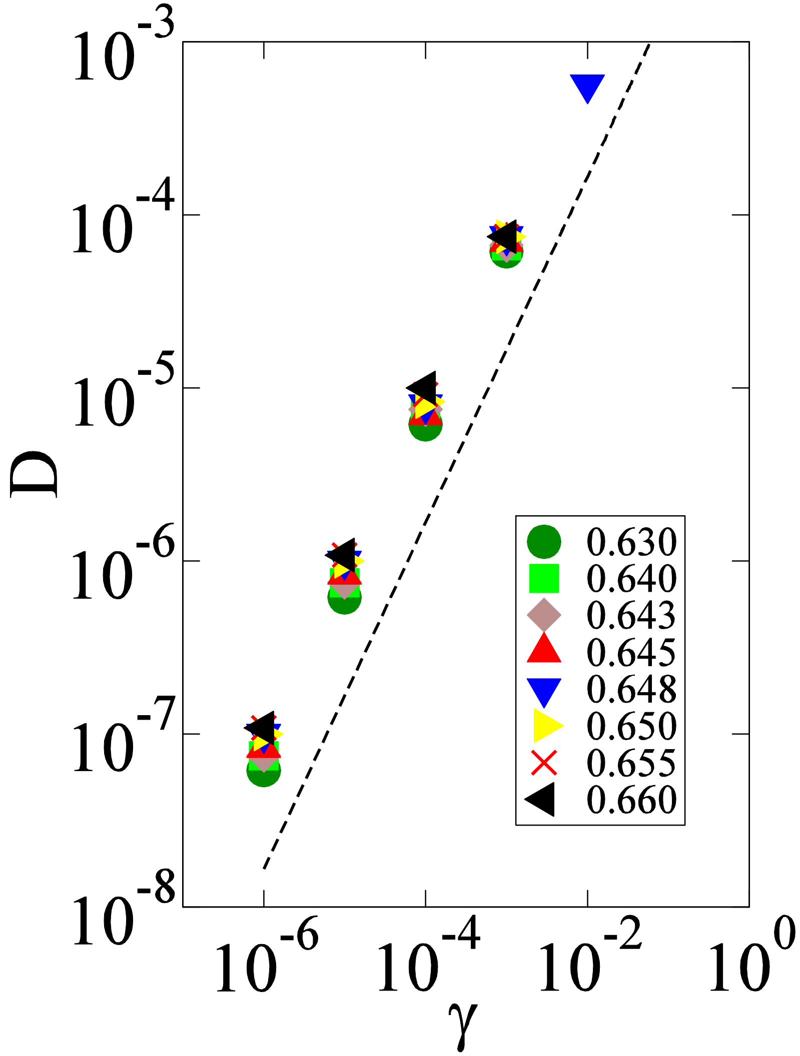
$$\langle \delta r^2(t) \rangle \approx T\tau_m{}^2 f(t/\tau_m)$$

$$T := \langle \delta v^2(0) \rangle$$

granular temperature

$$D \propto \tau_m T$$

diffusion coefficient vs shear rate

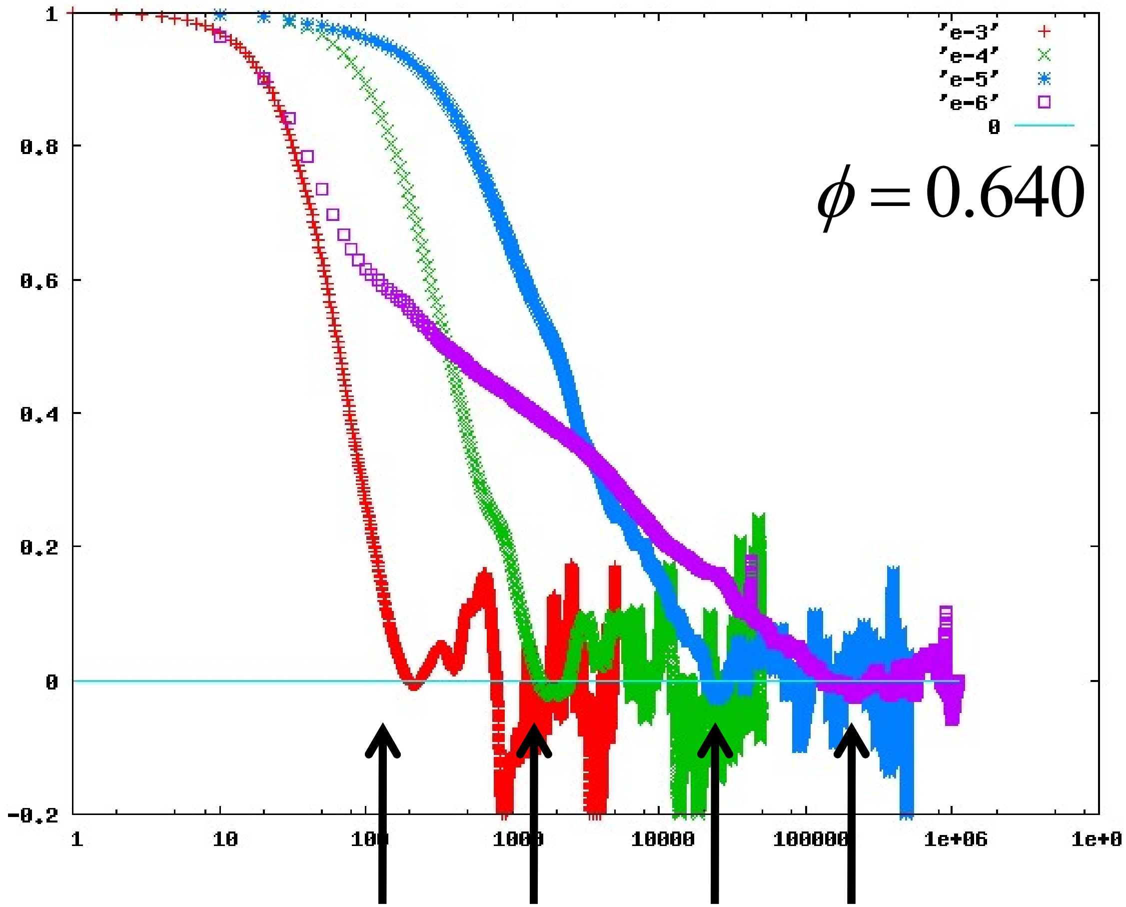


$$D \propto \dot{\gamma}$$

e.g. Brownian particles
(E. Weeks, this morning)

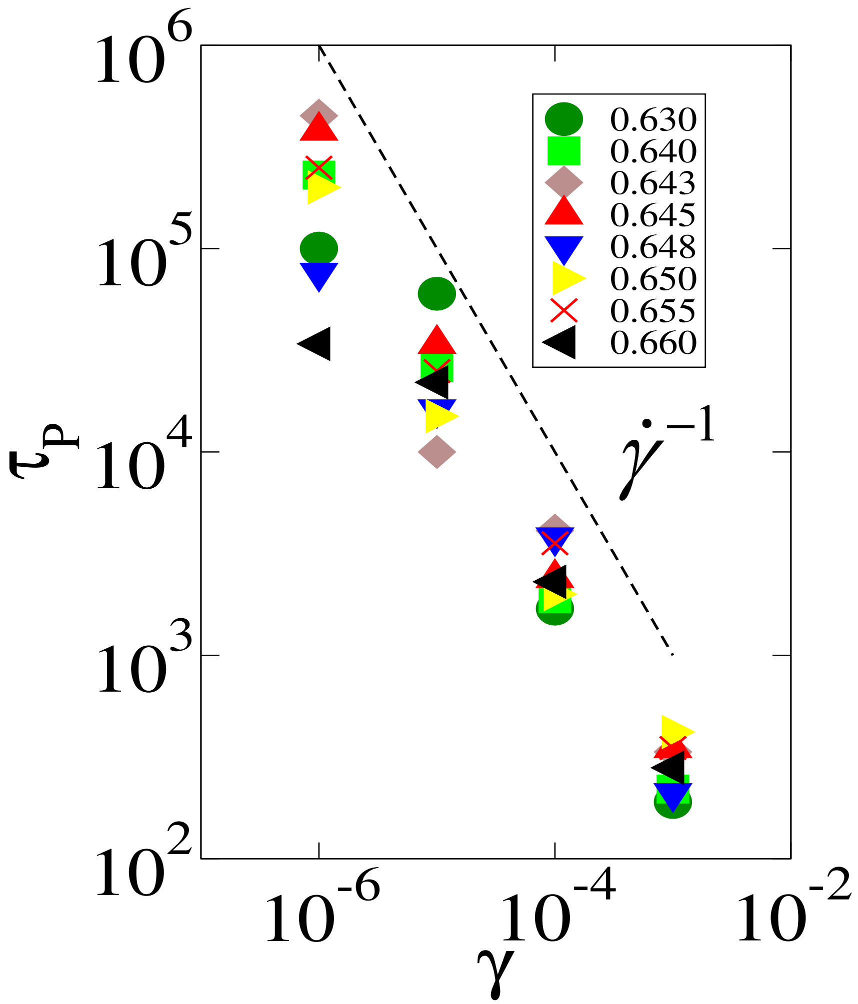
another time scale: pressure autocorrelation

$$C_P(t) := \langle \delta P(t) \delta P(0) \rangle / \langle \delta P^2 \rangle$$



define τ_P as $C_P(\tau_P) = 0$ relaxation time of pressure

pressure relaxation time



(structural relaxation time)

$$\tau_P \sim c\dot{\gamma}^{-1}$$

- rather trivial time scale
- weak dependence on density
- c is characteristic strain (depends on shear rate)

$c \sim 0.2$ (high shear rate)

$c \leq 0.1$ (lower shear rate)

NB. we obtain the same result for stress-stress corrrelation function.

partial summary

1. characteristic length of non-affine response

$$\xi \sim \dot{\gamma}^{-y_\gamma}$$ ξ ~ L → yield stress

2. two time scales of grain kinetics

A. non-affine response (spatially heterogeneous)

→ obeys scaling law $\tau_m = |\Phi|^{-\zeta} g_\pm\left(\frac{\dot{\gamma}}{|\Phi|^\Delta}\right)$

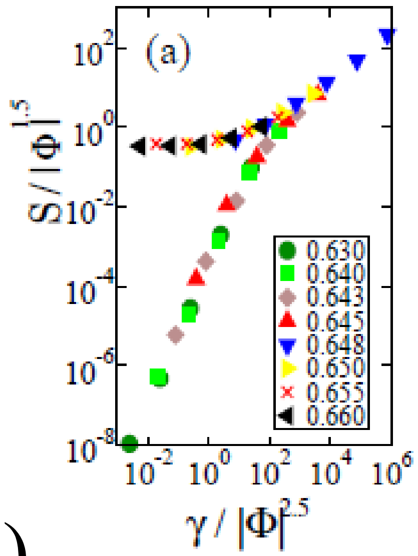
B. relaxation time of pressure & stress

→rather trivial $\tau_P \sim c\dot{\gamma}^{-1}$

C. time scale for shear thinning? $\dot{\gamma}_{th} \sim |\Phi|^{x_\Phi / x_\gamma} \sim \tau^{-1}$

characteristic time for shear thinning?

$$S(\Phi,\dot{\gamma}) = |\Phi|^{x_\Phi} f_\pm\left(\frac{\dot{\gamma}}{|\Phi|^{x_\Phi/x_\gamma}}\right)$$



$$\tau \sim |\Phi|^{-x_\Phi/x_\gamma} \sim \dot{\gamma}_{th}^{-1}$$

system with inertia $x_\Phi / x_\gamma = 2.5(2)$

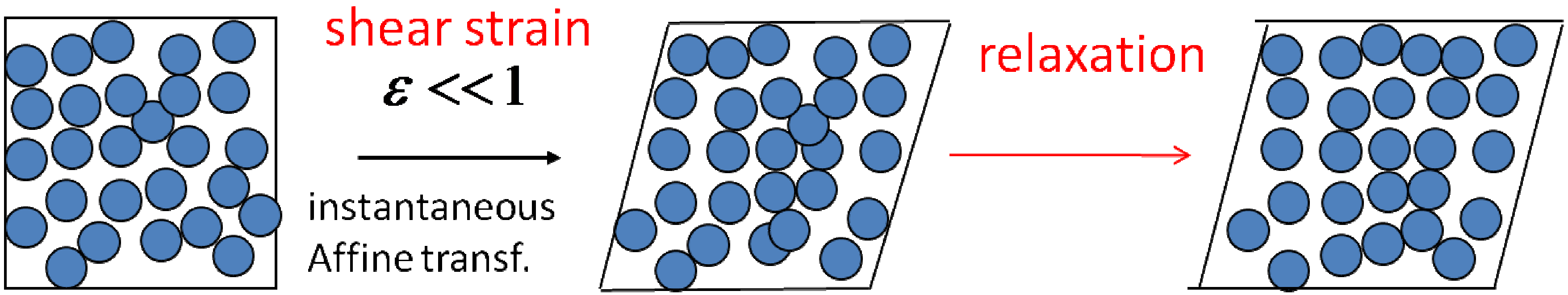
(Otsuki & Hayakawa 2009)

system without inertia $x_\Phi / x_\gamma = 3.0(2)$

(Olsson & Teitel 2007)

can show $\tau \sim |\Phi|^{-3.0}$ using structural relaxation time

relaxation from stepwise shear strain



E = 0
(S = 0)

E > 0
(S > 0)

E = 0
(S = 0)

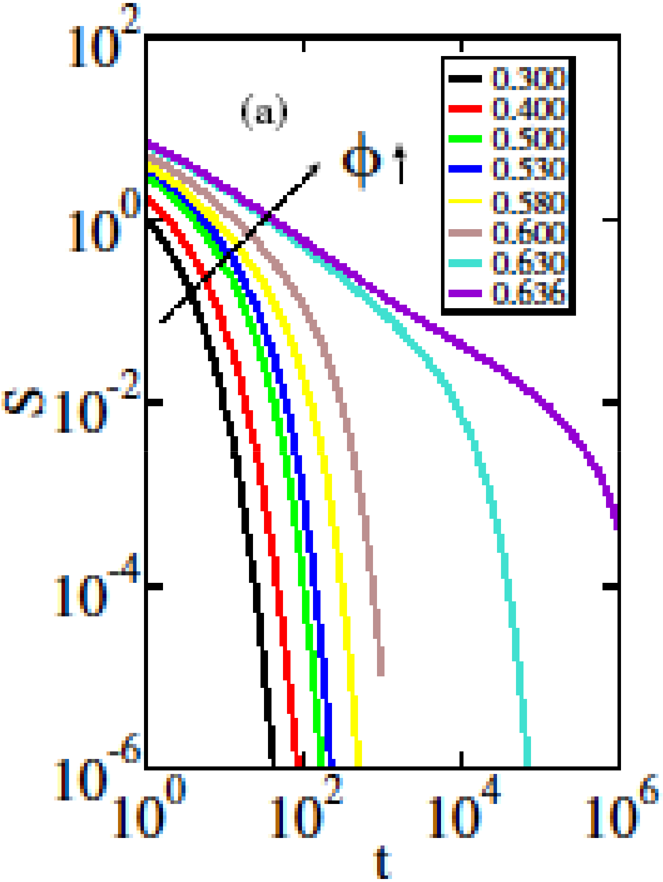
S: shear stress

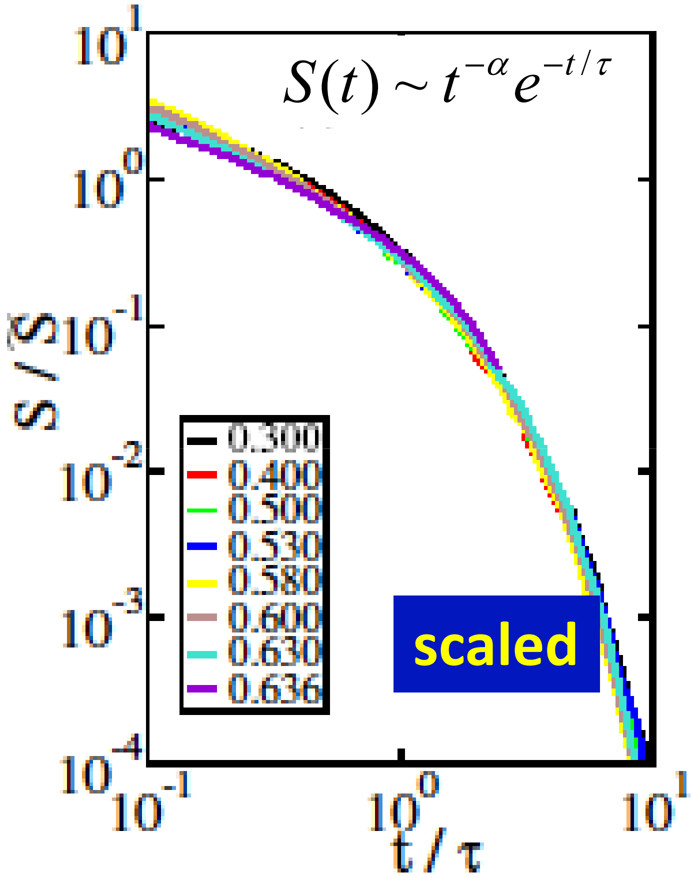
NOTE: 1. unjammed system $\phi < \phi_J$

2. overdamp dynamics $\dot{r}_i = -\frac{\partial E(\{r\})}{\partial r_i}$

shear stress relaxation

slower relaxation for denser system

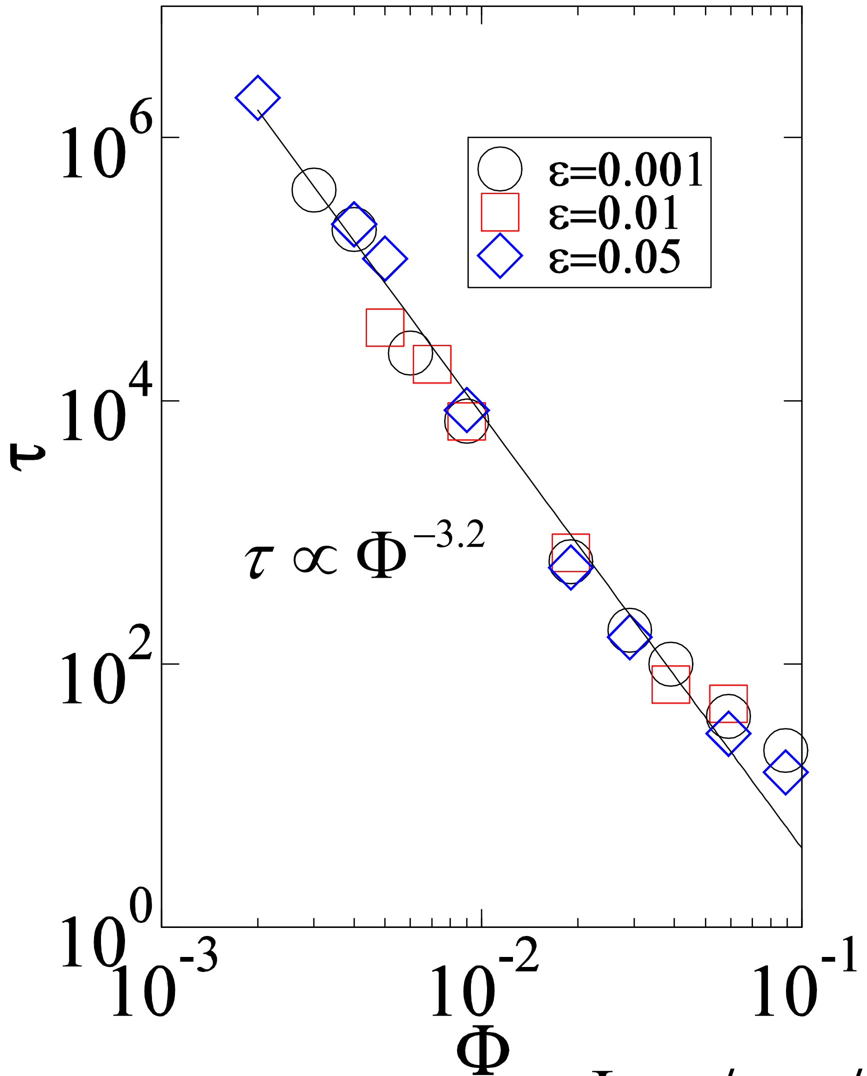




(TH, PRE 2008)

extracting time scale

structural relaxation time: 3D



$\Phi \equiv \phi_J - \phi$

$$\tau \sim \left(\phi_J - \phi\right)^{-\varsigma}$$

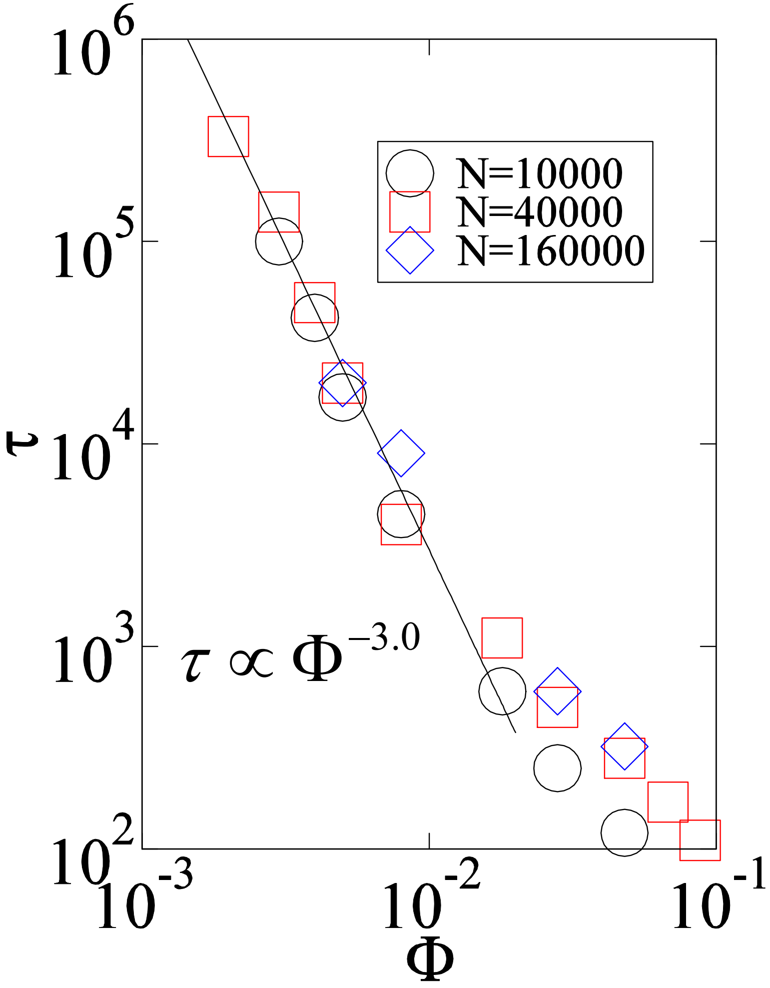
$$\varsigma = 3.2 \pm 0.3$$

$$\phi_J = 0.639$$

- independent of initial strain
- irrespective of system size

(TH, PRE 2008)

structural relaxation time: 2D



$$\tau \sim \left(\phi_J - \phi\right)^{-\varsigma}$$

$$\varsigma = 3.0 \pm 0.2$$

$$\varphi_J = 0.848$$

- irrespective of system size

outlook: characteristic time scale for shear thinning

$$\tau \sim \left|\Phi\right|^{-x_\Phi / x_\gamma} \sim \dot{\gamma}_{th}^{\ -1}$$

is structural relaxation time

(but not detectable in steady shear flow)

cf. in systems with inertia?

cf. in thermal systems $\dot{\gamma}_{th}^{\ -1} \approx \left(\frac{\partial \eta}{\partial P}\right)_T$ (Furukawa & Tanaka 2006)

relation between thermal and athermal systems?