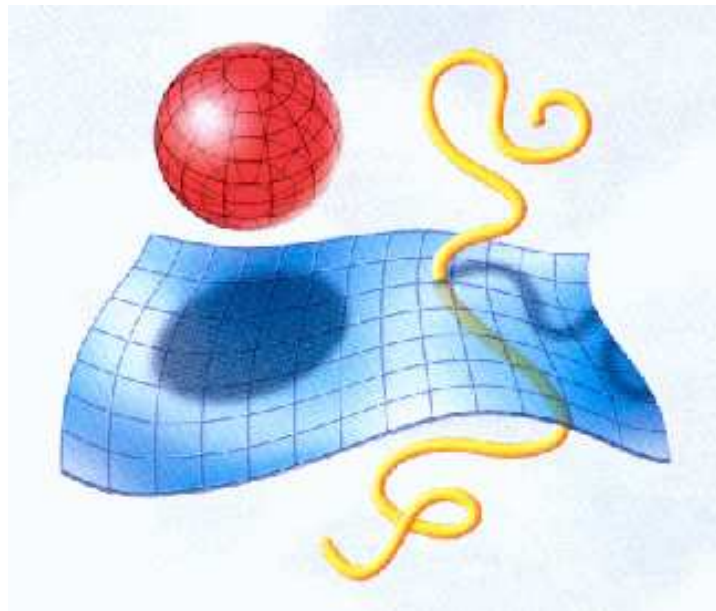


## Vesicles and Red Blood Cells in Microcapillary Flows

Gerhard Gompper and Hiroshi Noguchi\*

Institut für Festkörperforschung, and Institute for Advanced Simulations,  
Forschungszentrum Jülich, Germany

\* Institute for Solid State Physics, University of Tokyo, Japan

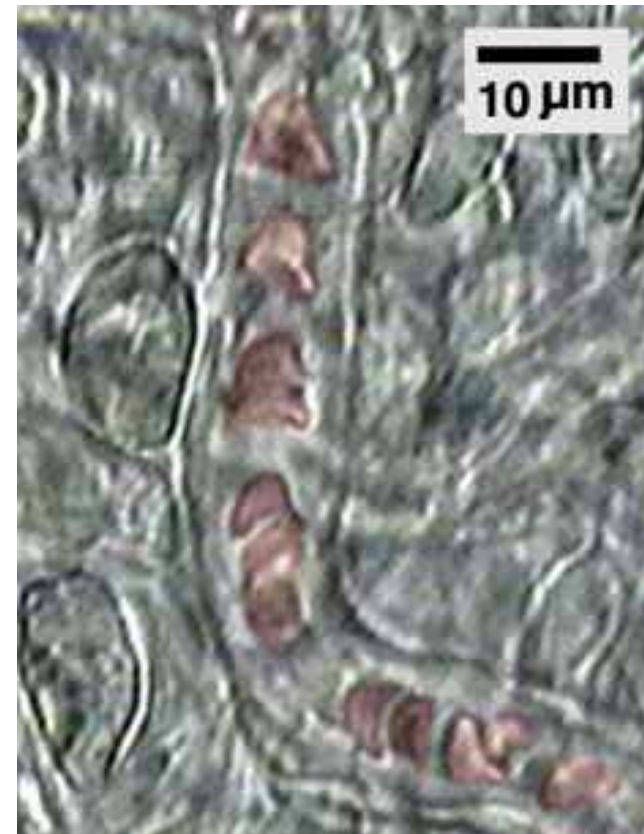


# Soft Matter Hydrodynamics

---

Cells and vesicles in flow:

- Red blood cells in microvessels:



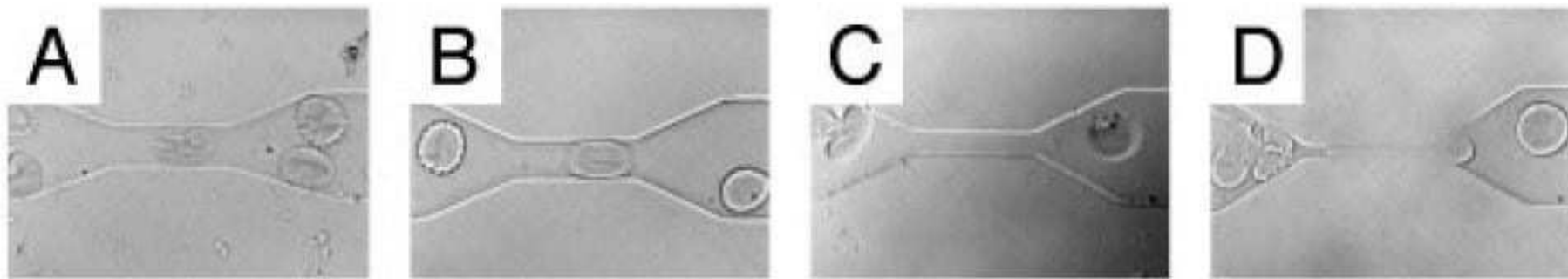
Diseases such as diabetes reduce deformability of red blood cells!

# Soft Matter Hydrodynamics

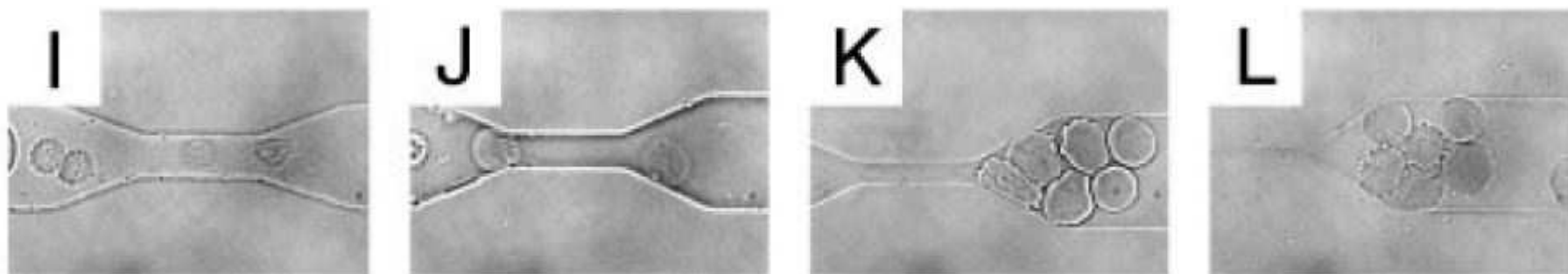
---

Example: Flow behavior of [malaria](#)-infected red blood cells in microchannels

Just after infection:



Late stage:



Diameter: 8  $\mu\text{m}$

6  $\mu\text{m}$

4  $\mu\text{m}$

2  $\mu\text{m}$

# Mesoscale Flow Simulations

---

Complex fluids: length- and time-scale gap between

- atomistic scale of solvent
- mesoscopic scale of dispersed particles (colloids, polymers, membranes)

→ **Mesoscale Simulation Techniques**

**Basic idea:**

- drastically simplify dynamics on molecular scale
- respect conservation laws for mass, momentum, energy

**Examples:**

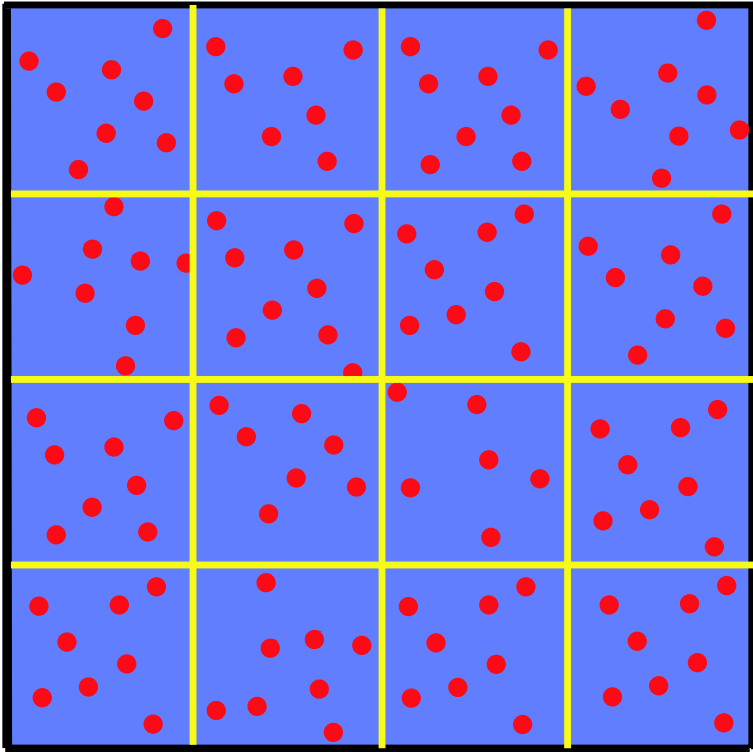
- Lattice Boltzmann Method (LBM)
- Dissipative Particle Dynamics (DPD)
- Multi-Particle-Collision Dynamics (MPC)

**Alternative approach:** Hydrodynamic interactions via Oseen tensor

# Mesoscale Hydrodynamics Simulations

---

## Multi-Particle-Collision Dynamics (MPC)



- coarse grained fluid
- point particles
- off-lattice method
- collisions inside “cells”
- thermal fluctuations

A. Malevanets and R. Kapral, J. Chem. Phys. 110 (1999)

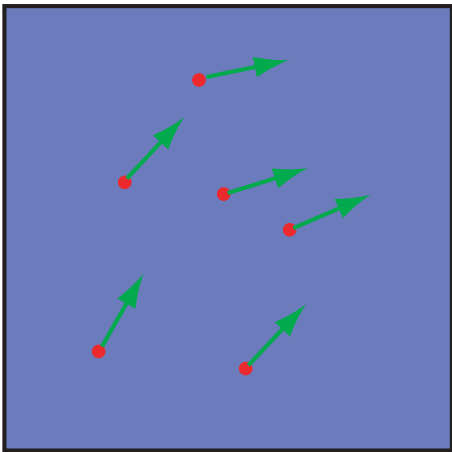
A. Malevanets and R. Kapral, J. Chem. Phys. 112 (2000)

# Multi-Particle Collision Dynamics (MPC)

---

Flow dynamics: Two step process

Streaming



- ballistic motion

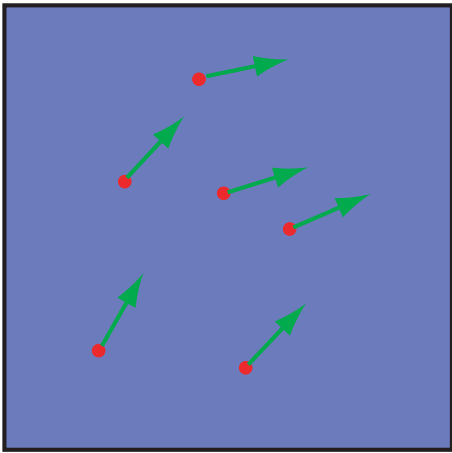
$$\mathbf{r}_i(t + h) = \mathbf{r}_i(t) + \mathbf{v}_i(t)h$$

# Multi-Particle Collision Dynamics (MPC)

---

Flow dynamics: Two step process

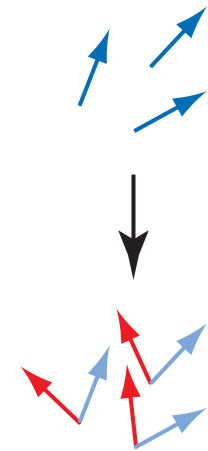
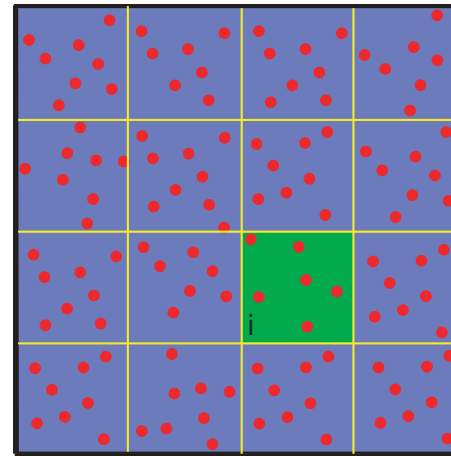
Streaming



- ballistic motion

$$\mathbf{r}_i(t + h) = \mathbf{r}_i(t) + \mathbf{v}_i(t)h$$

Collision



- mean velocity per cell

$$\bar{\mathbf{v}}_i(t) = \frac{1}{n_i} \sum_{j \in C_i} \mathbf{v}_j(t)$$

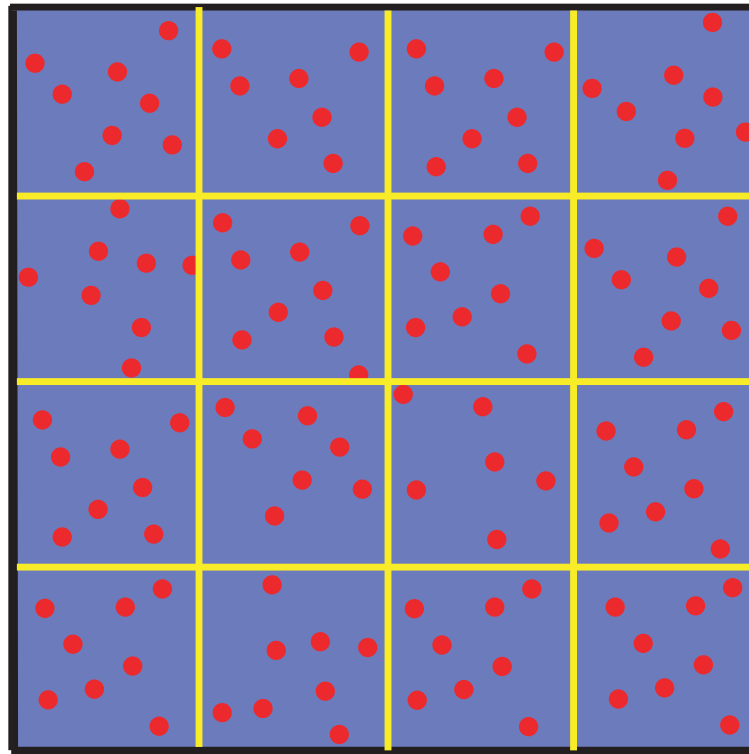
- rotation of relative velocity by angle  $\alpha$

$$\mathbf{v}'_i = \bar{\mathbf{v}}_i + \mathbf{D}(\alpha)(\mathbf{v}_i - \bar{\mathbf{v}}_i)$$

# Mesoscale Flow Simulations: MPC

---

- Lattice of collision cells: breakdown of **Galilean invariance**
- Restore Galilean invariance exactly: **random shifts** of cell lattice

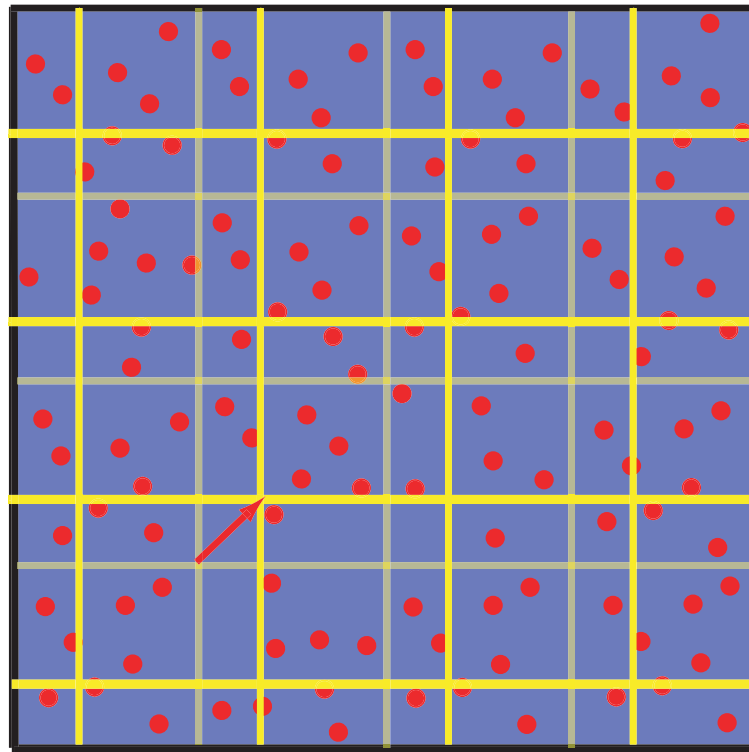




# Mesoscale Flow Simulations: MPC

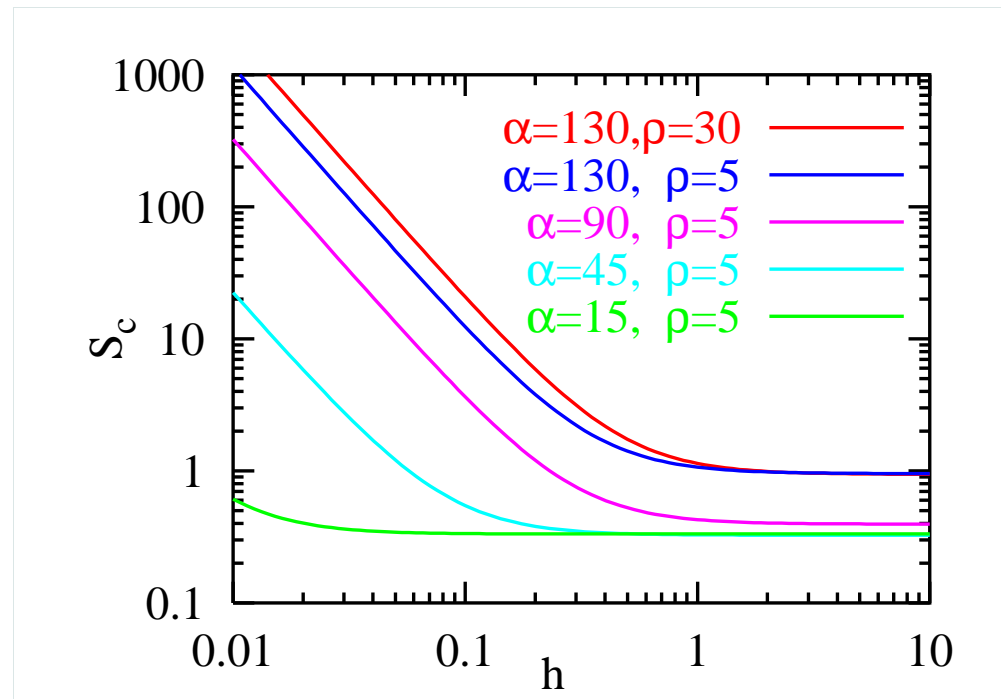
---

- Lattice of collision cells: breakdown of **Galilean invariance**
- Restore Galilean invariance exactly: **random shifts** of cell lattice



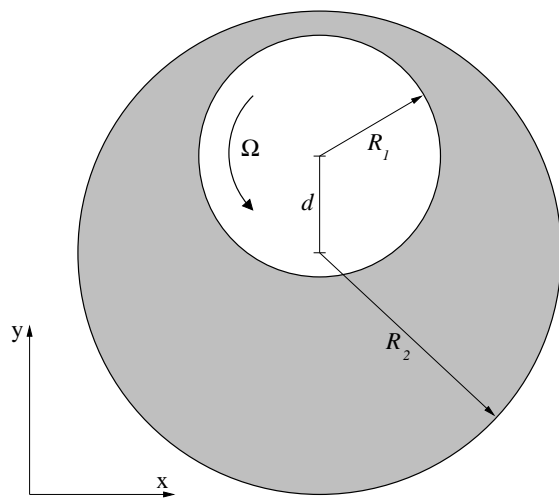
# Low-Reynolds-Number Hydrodynamics

- **Reynolds number**  $Re = v_{\max}L/\nu \sim$  inertia forces / friction forces  
For soft matter systems with characteristic length scales of  $\mu m$ :  
 $Re \simeq 10^{-3}$
- **Schmidt number**  $Sc = \nu/D \sim$  momentum transp. / mass transp.  
Gases:  $Sc \simeq 1$ , liquids:  $Sc \simeq 10^3$

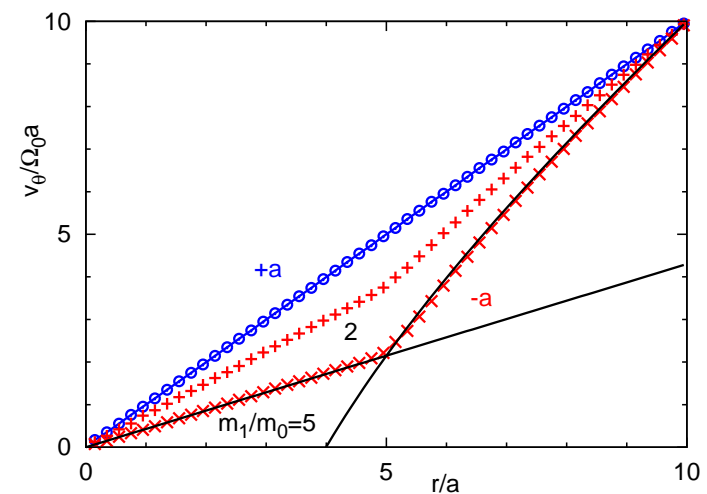


# Other MPC Methods

- Anderson thermostat (MPC-AT-a):  
Choose new relative velocities from Maxwell-Boltzmann distribution
- Angular-momentum conservation (MPC-AT+a):  
Modify collision rule to conserve angular momentum
- Importance of angular-momentum conservation:  
**Rotating fluid drop** with different viscosity in circular Couette flow



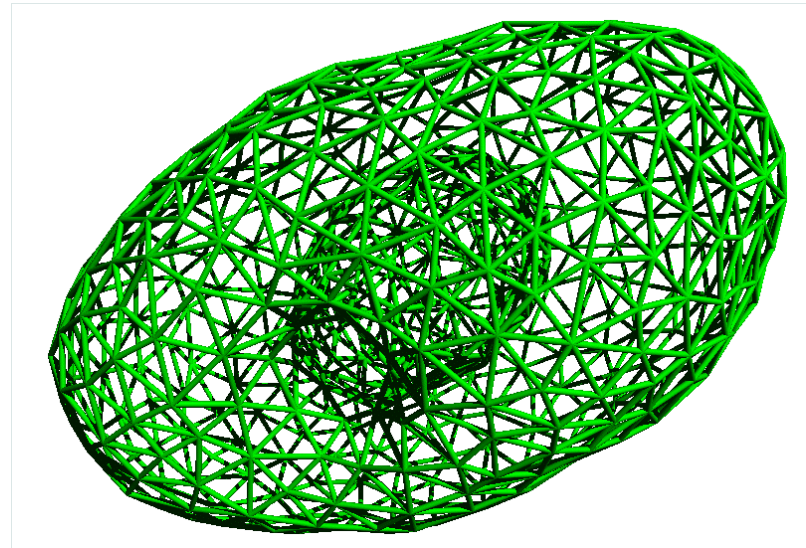
Angular  
velocity  
profile:



# Membranes

---

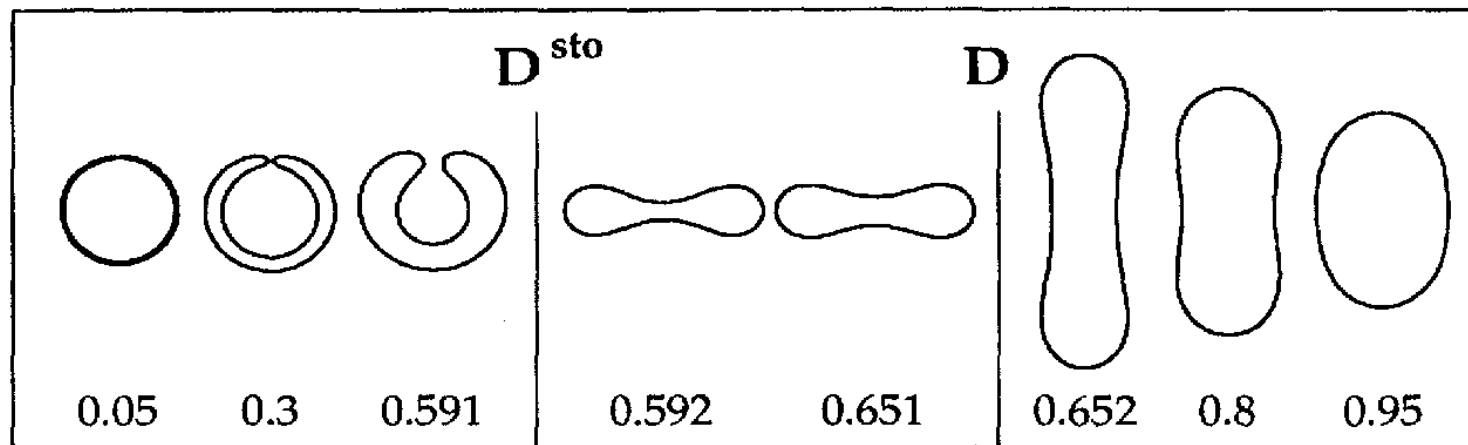
## Hydrodynamics of Membranes and Vesicles



# Equilibrium Vesicle Shapes

---

Minimize curvature energy for fixed area  $A = 4\pi R_0^2$  and reduced volume  $V^* = V/V_0$ , where  $V_0 = 4\pi R_0^3/3$ :



stomatocyte

discoctyte

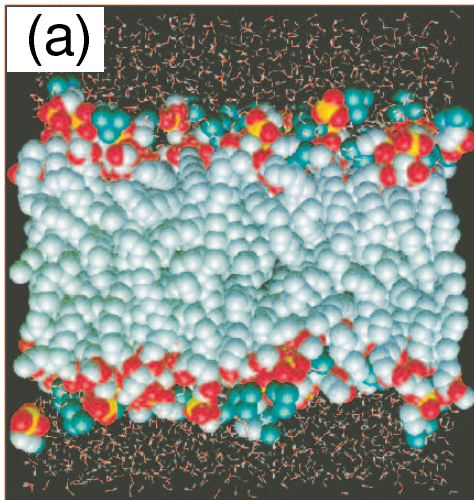
prolate

U. Seifert, K. Berndl, and R. Lipowsky, Phys. Rev. A 44 (1991)

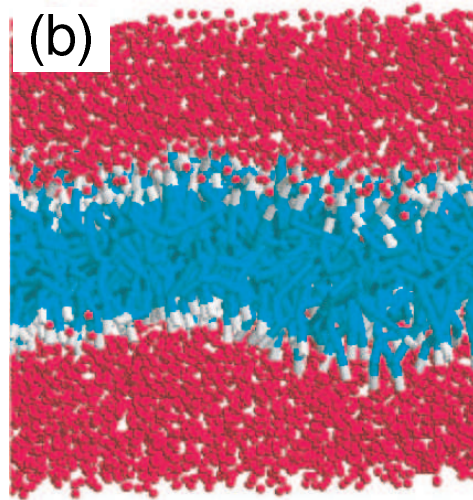
# Simulations of Membranes

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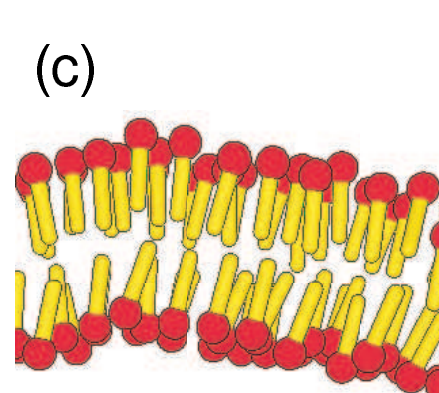
Modelling of membranes on different length scales:



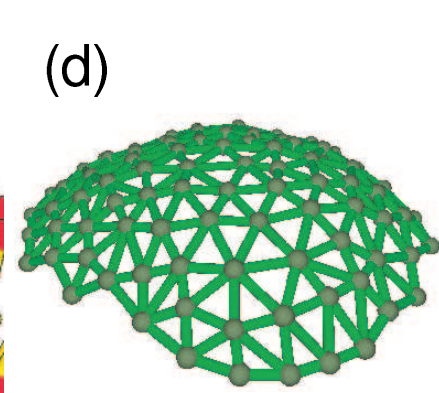
atomistic



coarse-grained



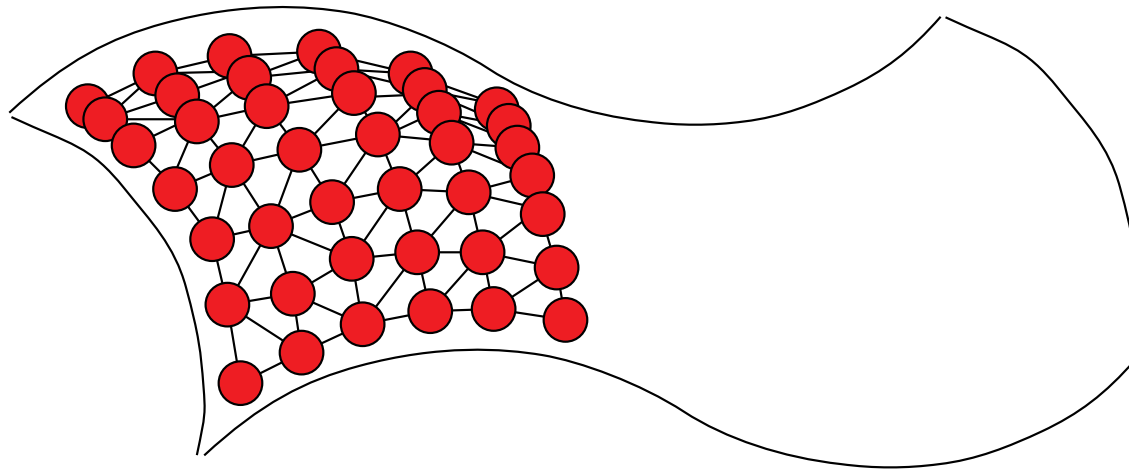
solvent-free



triangulated

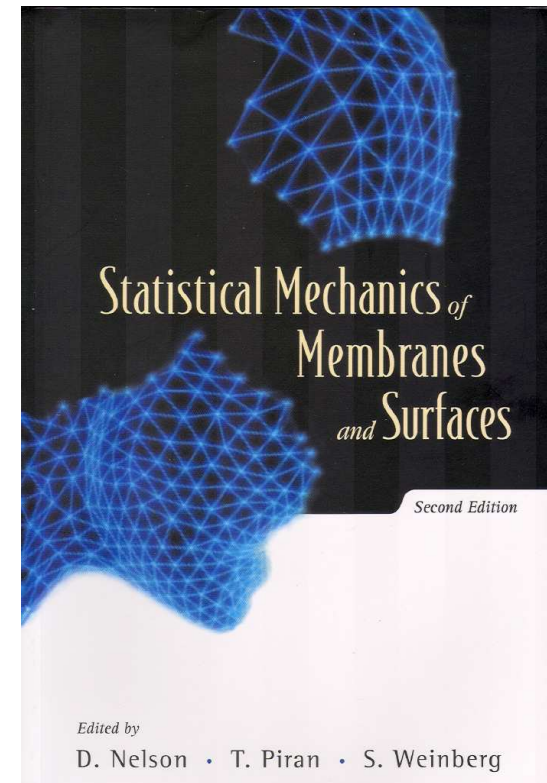
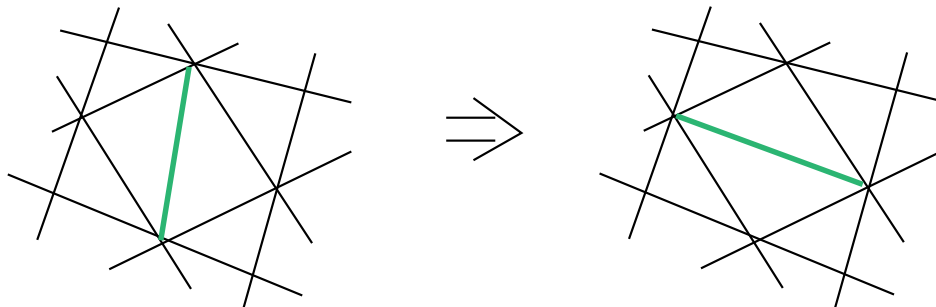
# Simulations of Membranes

## Dynamically triangulated surfaces



Hard-core diameter  $\sigma$   
Tether length  $L$ :  $\sigma < L < \sqrt{3}\sigma$   
--> self-avoidance

Dynamic triangulation:



G. Gompper & D.M. Kroll (2004)

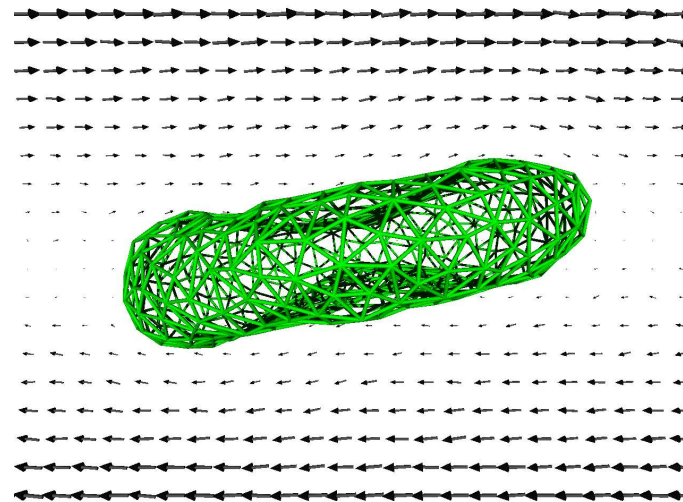
# Membrane Hydrodynamics

---

Interaction between membrane and fluid:

- Streaming step:  
bounce-back scattering of solvent particles on triangles
- Collision step:  
membrane vertices are included in MPC collisions

implies **impenetrable membrane**  
with **no-slip boundary conditions**.



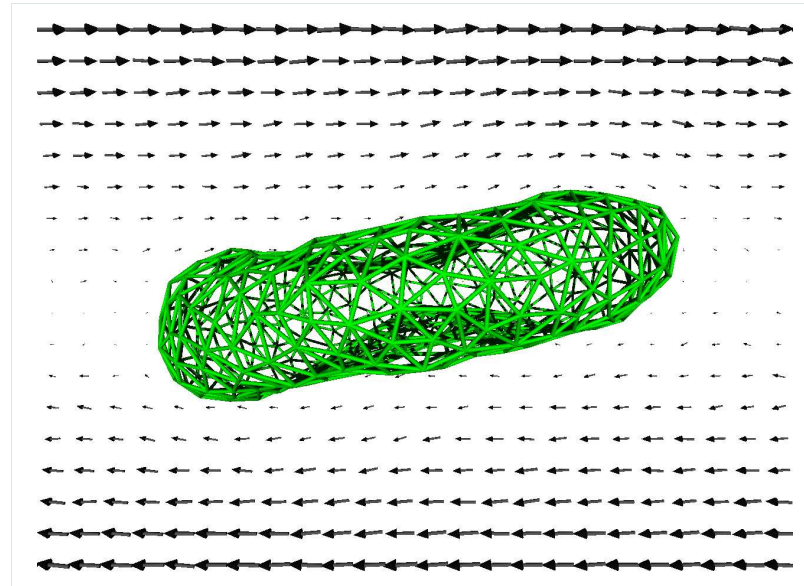
H. Noguchi and G. Gompper, Phys. Rev. Lett. **93** (2004); Phys. Rev. E **72** (2005)



# Membrane Hydrodynamics

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## Vesicle Dynamics in Shear Flow



# Vesicles in Shear Flow

---

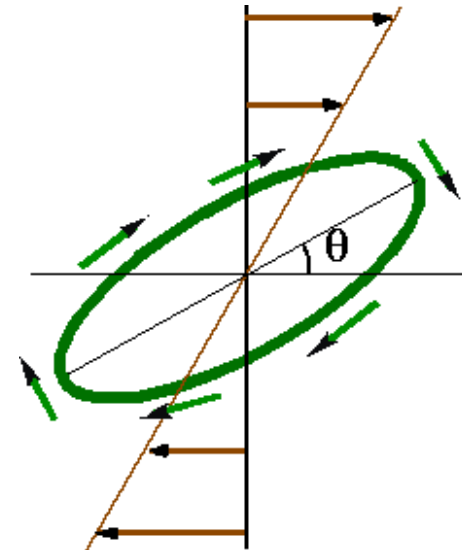
Parameter: shear rate  $\dot{\gamma}$

Variables:

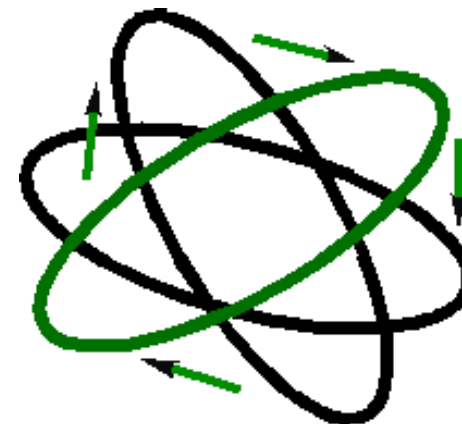
- Reduced volume  $V^*$
- Shape
- Membrane viscosity  $\eta_{mb}$
- Internal viscosity  $\eta_{in}$

Behavior in shear flow:

- Tank-treading
- Tumbling
- Swinging (vacillating-breathing, trembling)
- Shape transformations

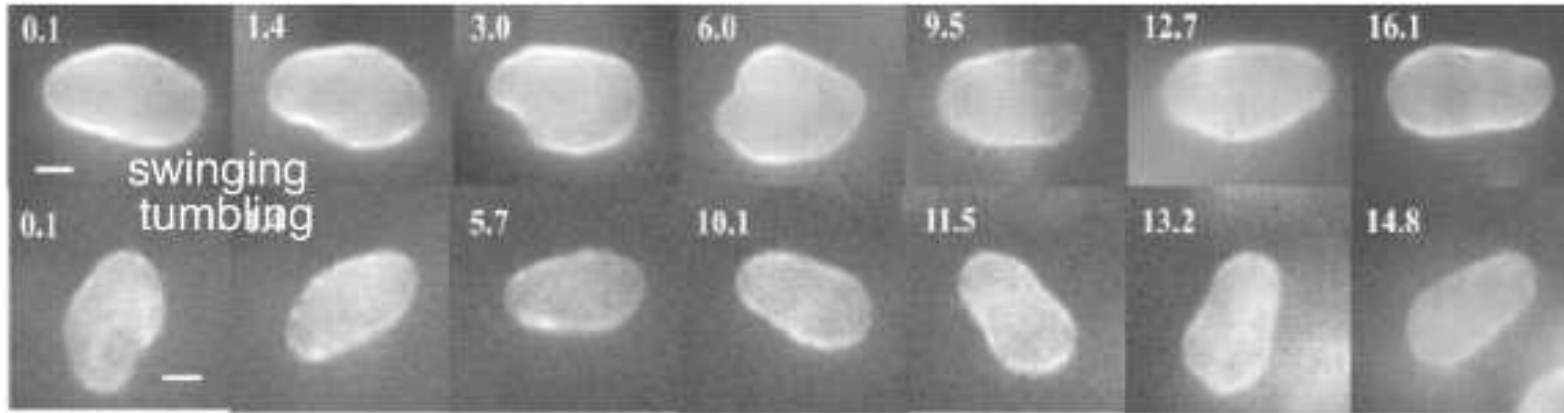


low viscosity



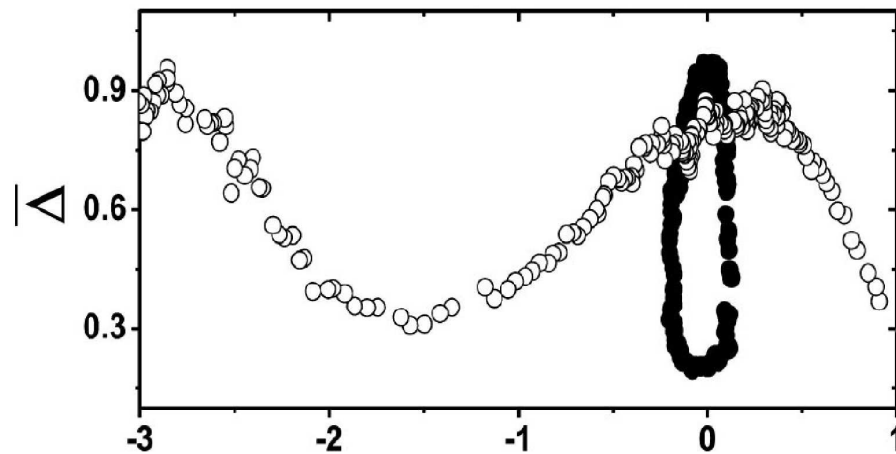
high viscosity

# Swinging, Vacillating-breathing, Trembling



$$\dot{\gamma} = 1.8s^{-1}$$

$$\dot{\gamma} = 1.7s^{-1}$$



inclination angle  $\theta$

Transition from tumbling to swinging with increasing shear rate  $\dot{\gamma}$

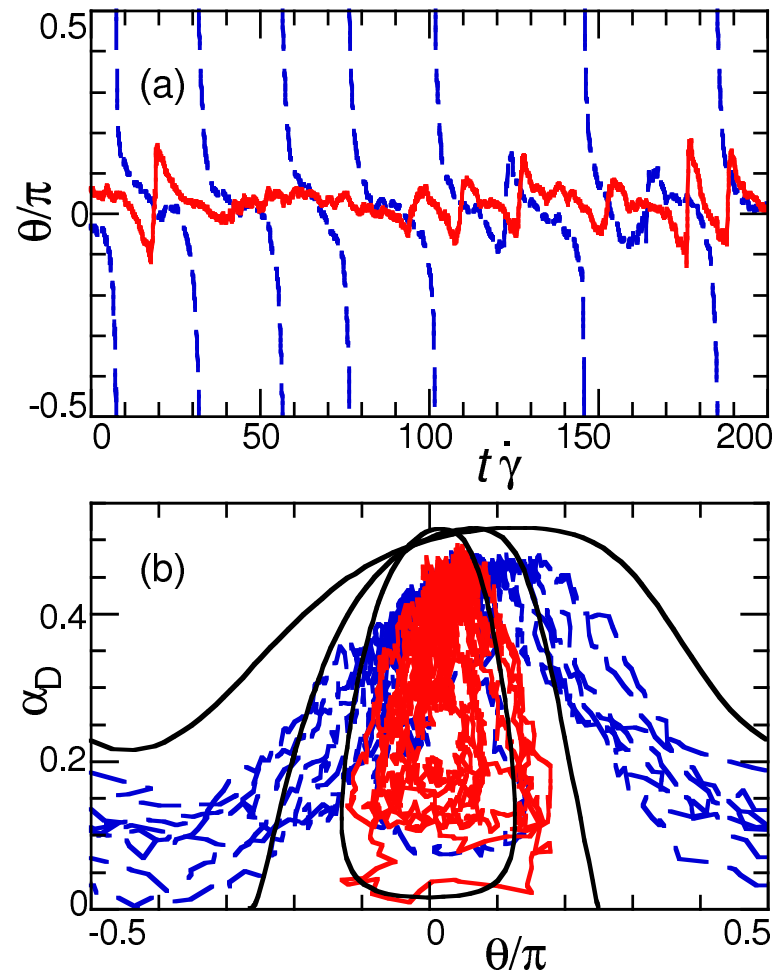
V. Kantsler and V. Steinberg,

Phys. Rev. Lett. **96** (2006)

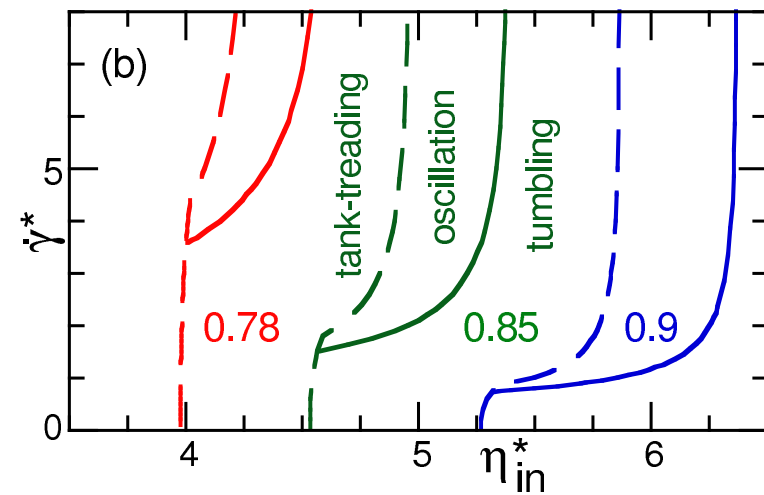
Theory: C. Misbah, Phys. Rev. Lett. **96** (2006); H. Noguchi and G. Gompper, Phys. Rev. Lett. **98** (2007); P.M. Vlahovska and R.S. Garcia, Phys. Rev. E **78** (2007); V.V. Lebedev et al., Phys. Rev. Lett. **99** (2007) ...

# Swinging of Fluid Vesicles: Theory

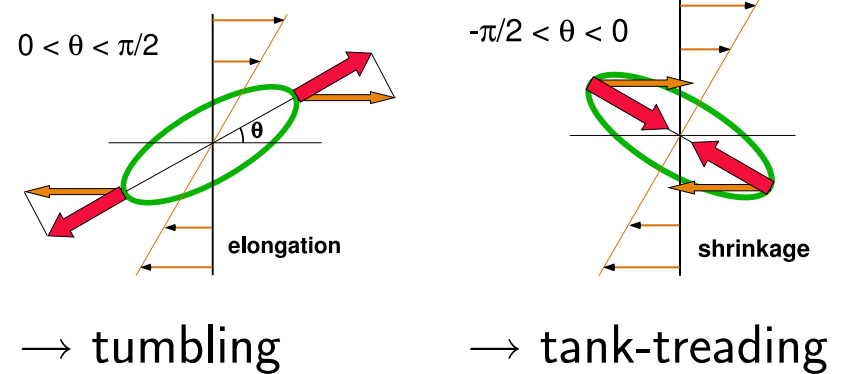
Shape dynamics:



Phase diagram:

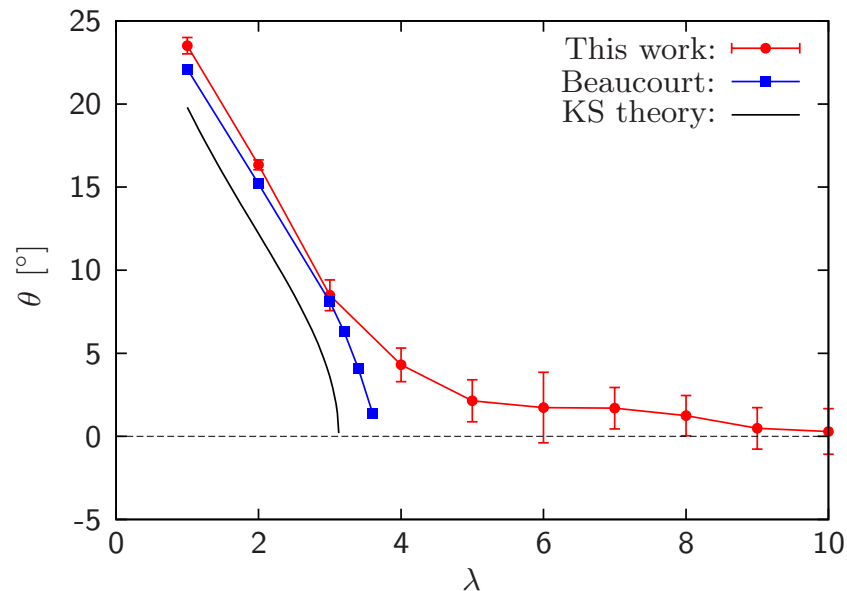


Mechanism:

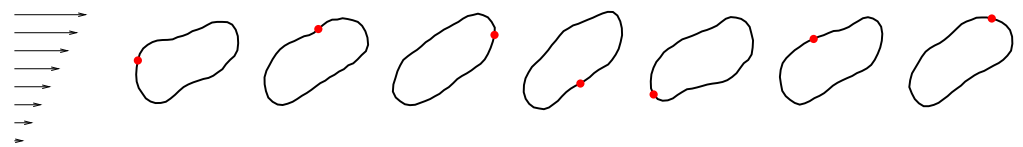


# Vesicles with Viscosity Contrast in Bulk

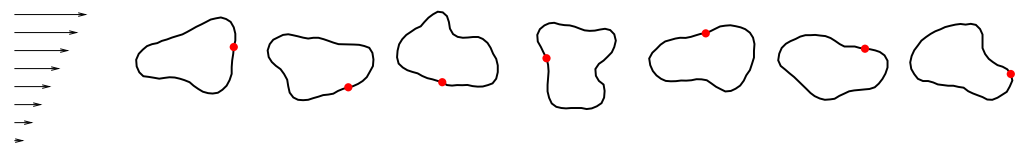
- Two-dimensional vesicles in shear flow
- Employ MPC-AT+a (with angular momentum conservation)
- Change viscosity contrast  $\lambda$  by varying mass of fluid particles



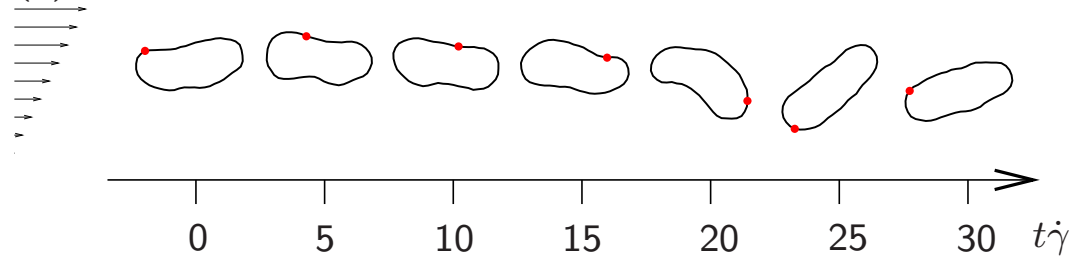
(a) Tank-treading:



(b) Swinging:

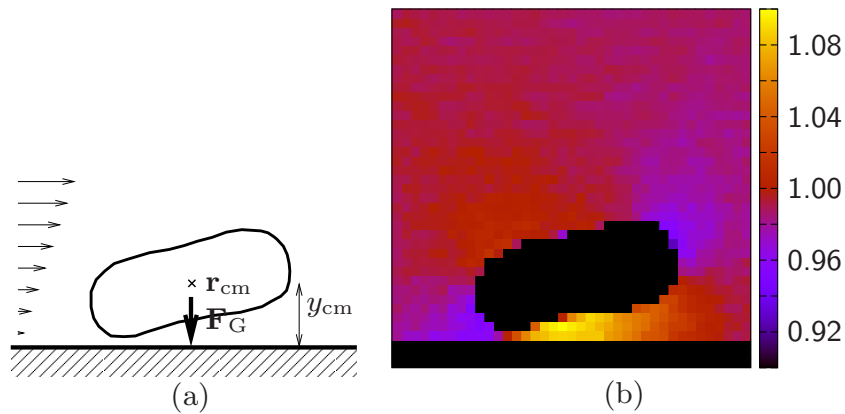


(c) Tumbling:



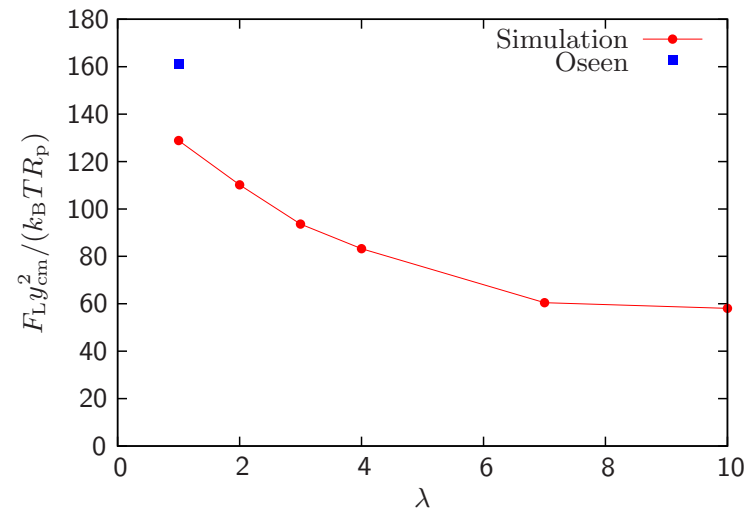
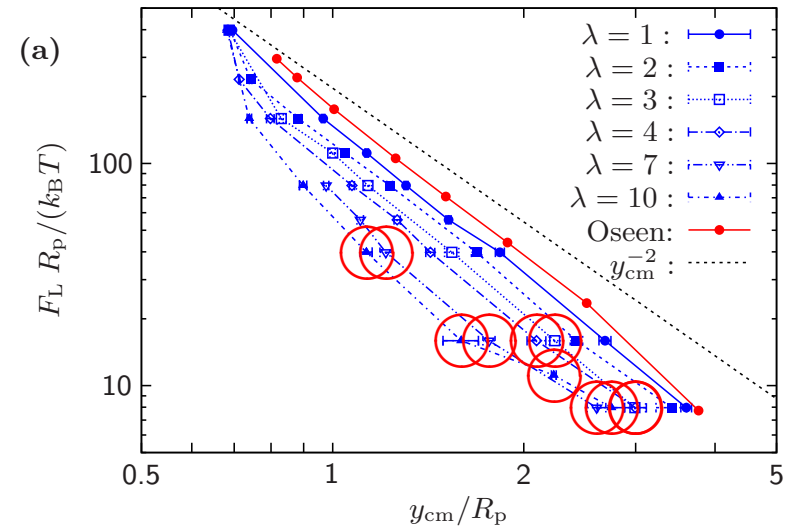
# Vesicles with Viscosity Contrast near Wall

Vesicle in gravitational field near wall:



Lift force  $F_L$  balanced by gravitational force  $F_G$

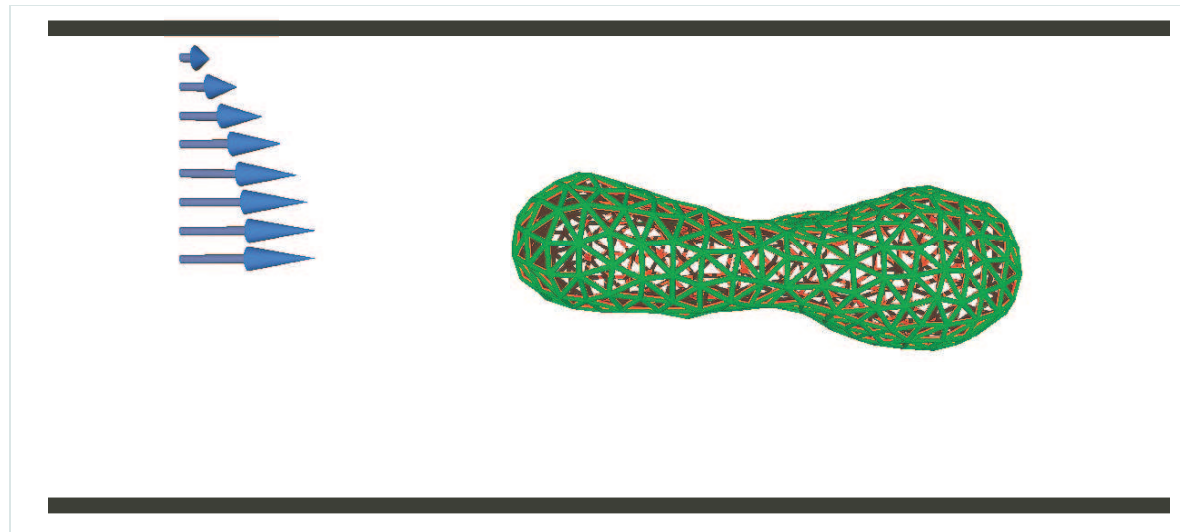
Lift force depends on viscosity contrast  $\lambda = \eta_{in}/\eta_{out}$



# Membrane Hydrodynamics

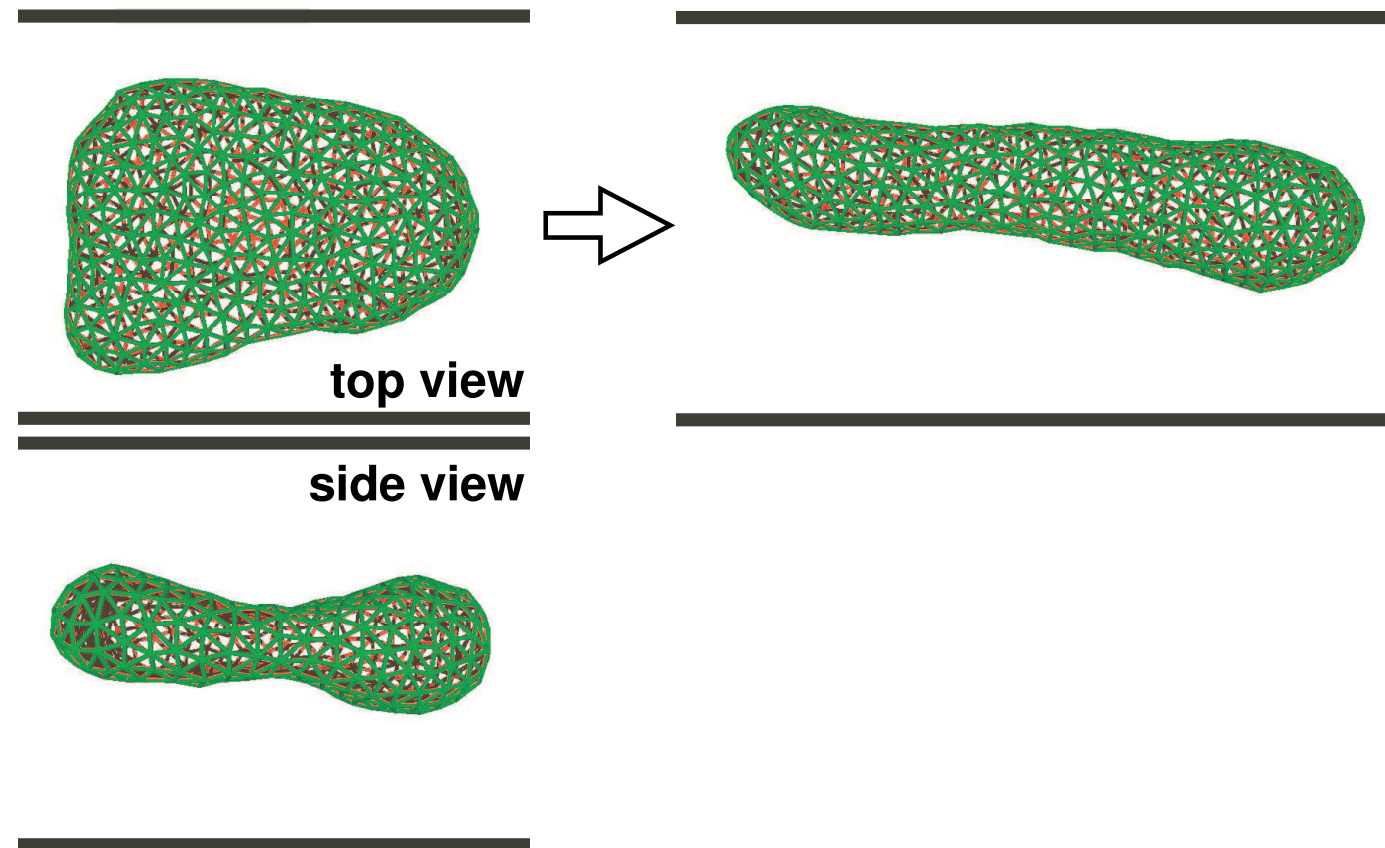
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## Vesicle and Cells in Capillary Flow



# Capillary Flow: Fluid Vesicles

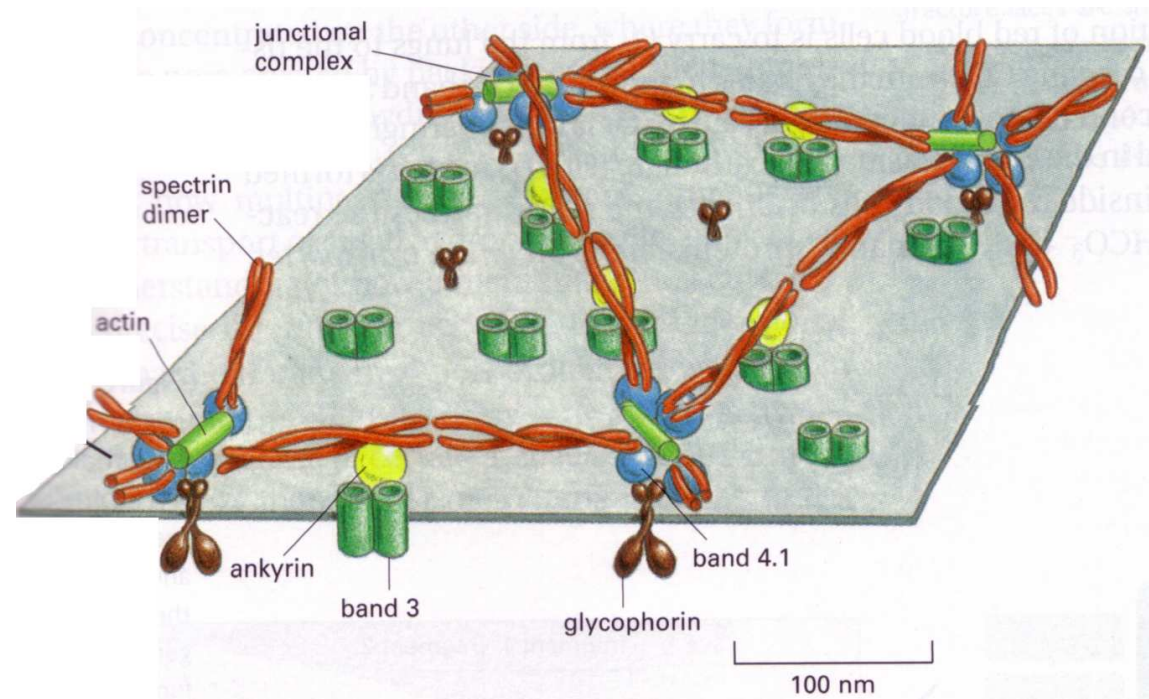
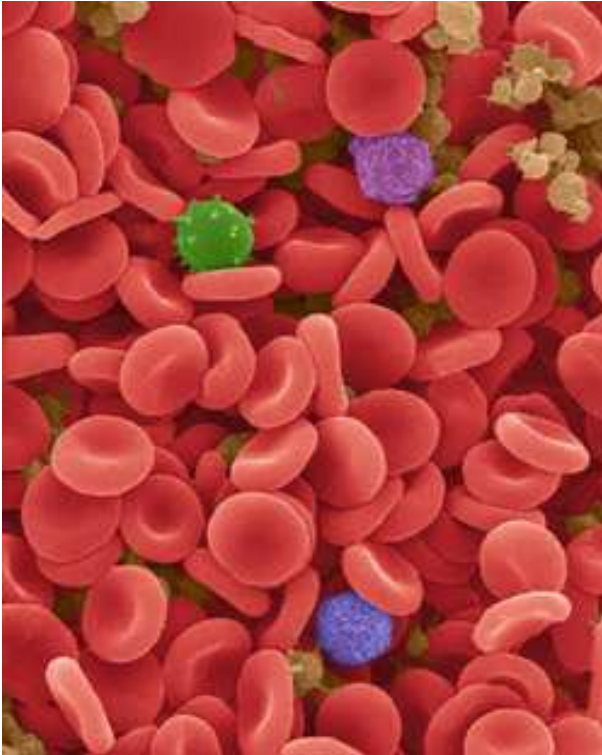
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- small flow velocities: vesicle axis **perpendicular** to capillary axis  $\longrightarrow$  **no axial symmetry!**
- discocyte-to-prolate transition with increasing flow



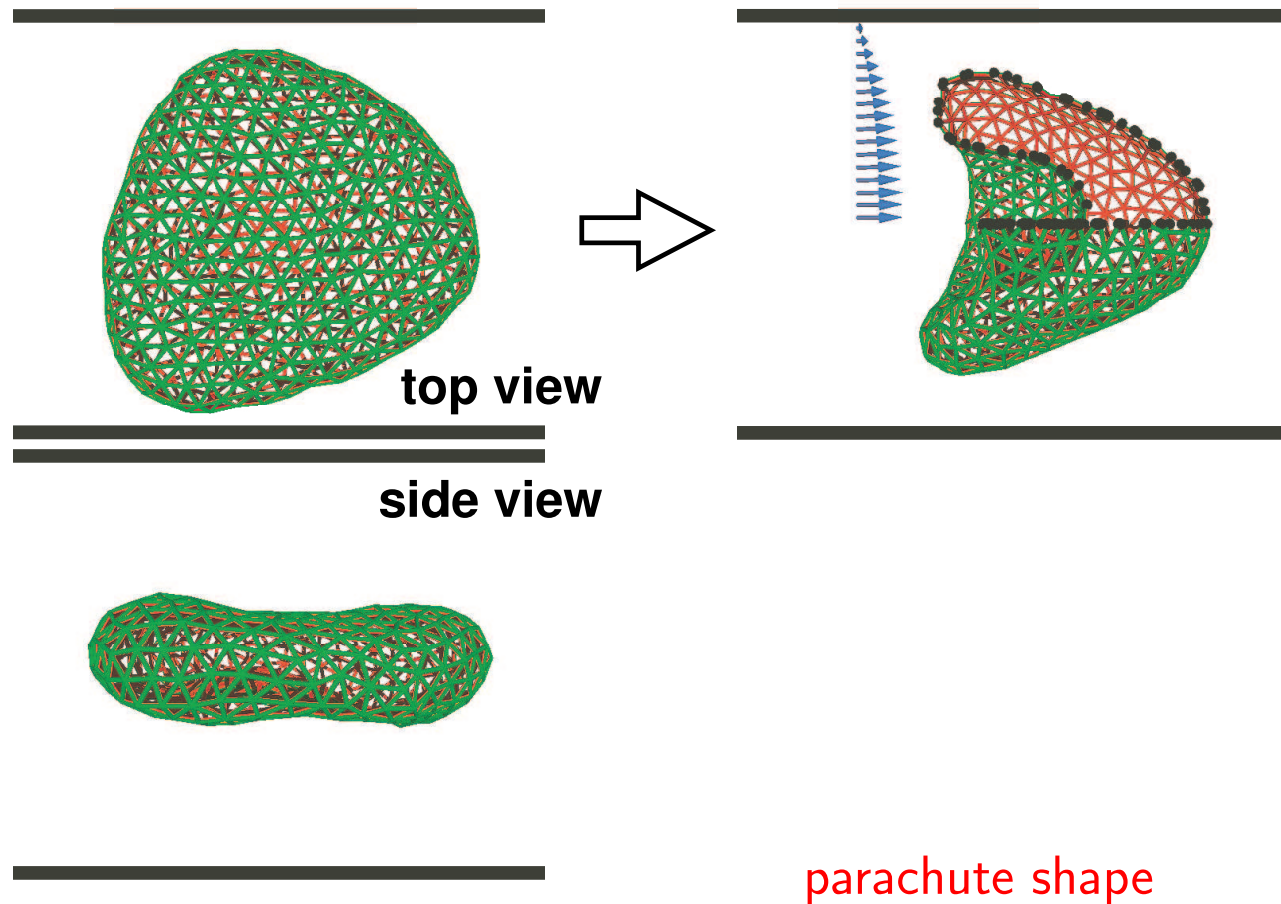
# Capillary Flow: Red Blood Cells



- Spectrin network induces shear elasticity  $\mu$  of composite membrane
- Elastic parameters:  $\kappa/k_B T = 50$ ,  $\mu R_0^2/k_B T = 5000$

# Capillary Flow: Elastic Vesicles

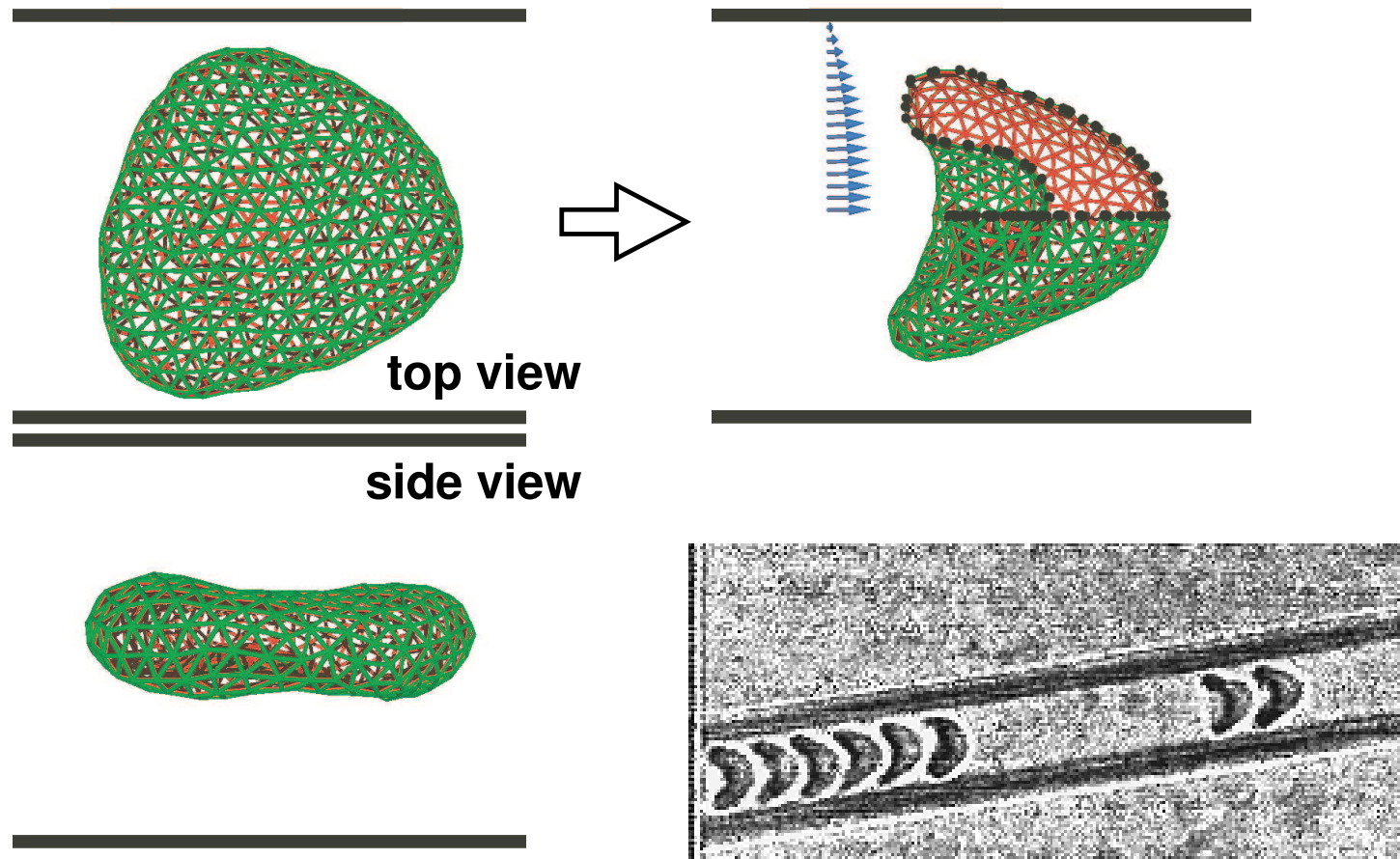
- Elastic vesicle:
- curvature and shear elasticity ( $\kappa = 20 k_B T$ ,  $\mu = 110 k_B T / R_0^2$ )
  - model for red blood cells



# Capillary Flow: Elastic Vesicles

---

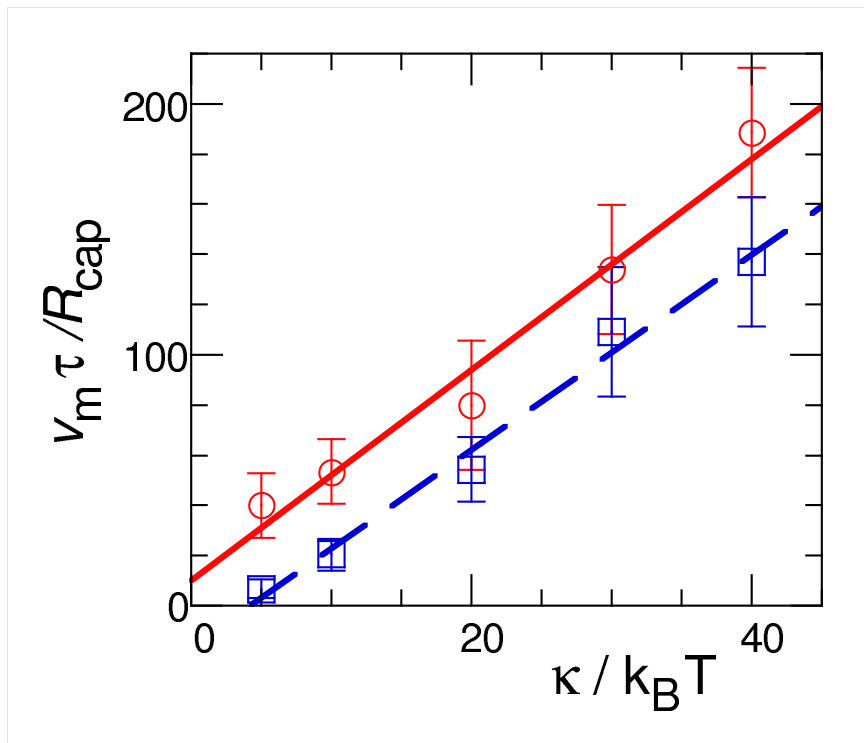
- Elastic vesicle:
- curvature and shear elasticity
  - model for red blood cells



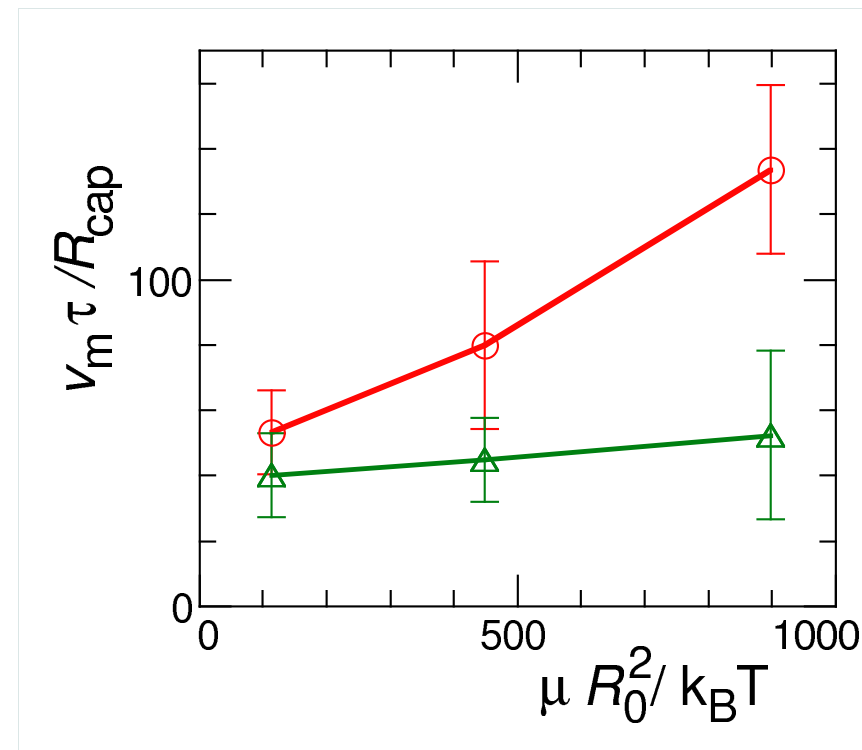
# Capillary Flow: Red Blood Cells

Shear elasticity suppresses prolate shapes (large deformations)

Flow velocity at discocyte-to-parachute transition



bending rigidity



shear modulus

Implies for RBCs:  $v_{trans} \simeq 0.2 \text{ mm/s}$  for  $R_{cap} = 4.6 \mu\text{m}$

# RBC Clustering & Alignment in Flow

---

Physiological conditions: Hematocrit (volume fraction of RBCs)  $H = 0.45$

Lower in narrow capillaries  $H_T = 0.1...0.2$

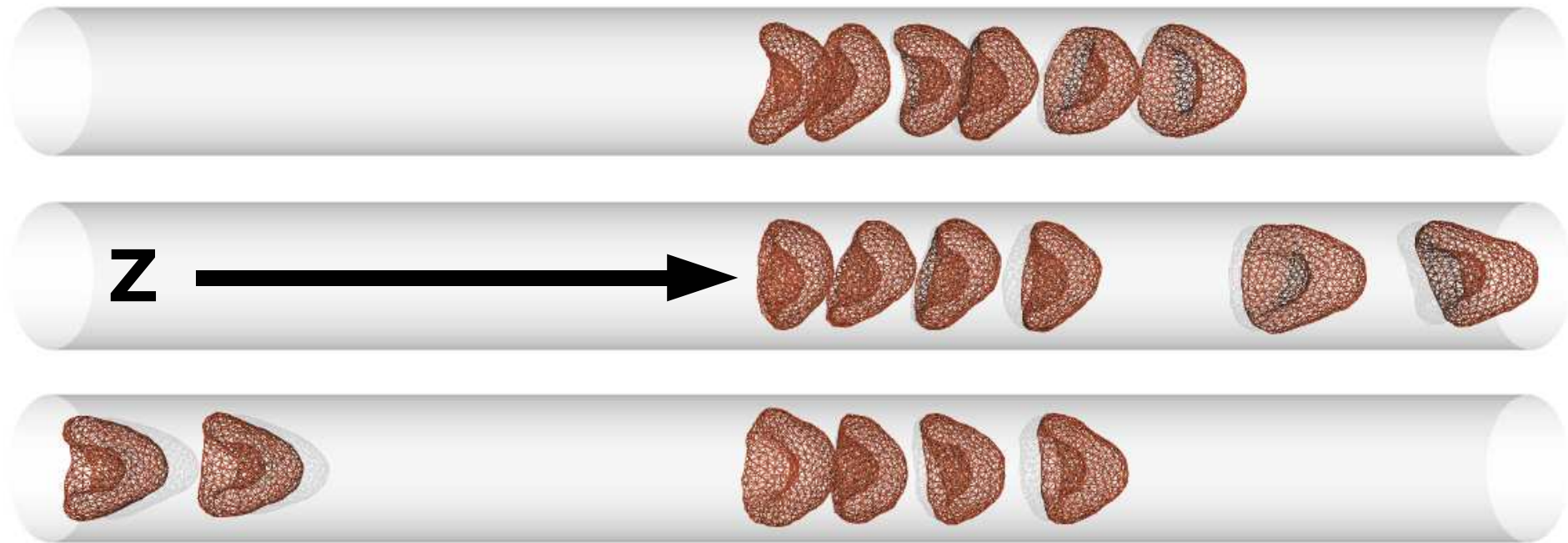
Therefore: Hydrodynamic interactions between RBCs very important

Note: No direct attractive interactions considered!

# RBC Clustering & Alignment in Flow

---

Low hematocrit  $H_T$ :

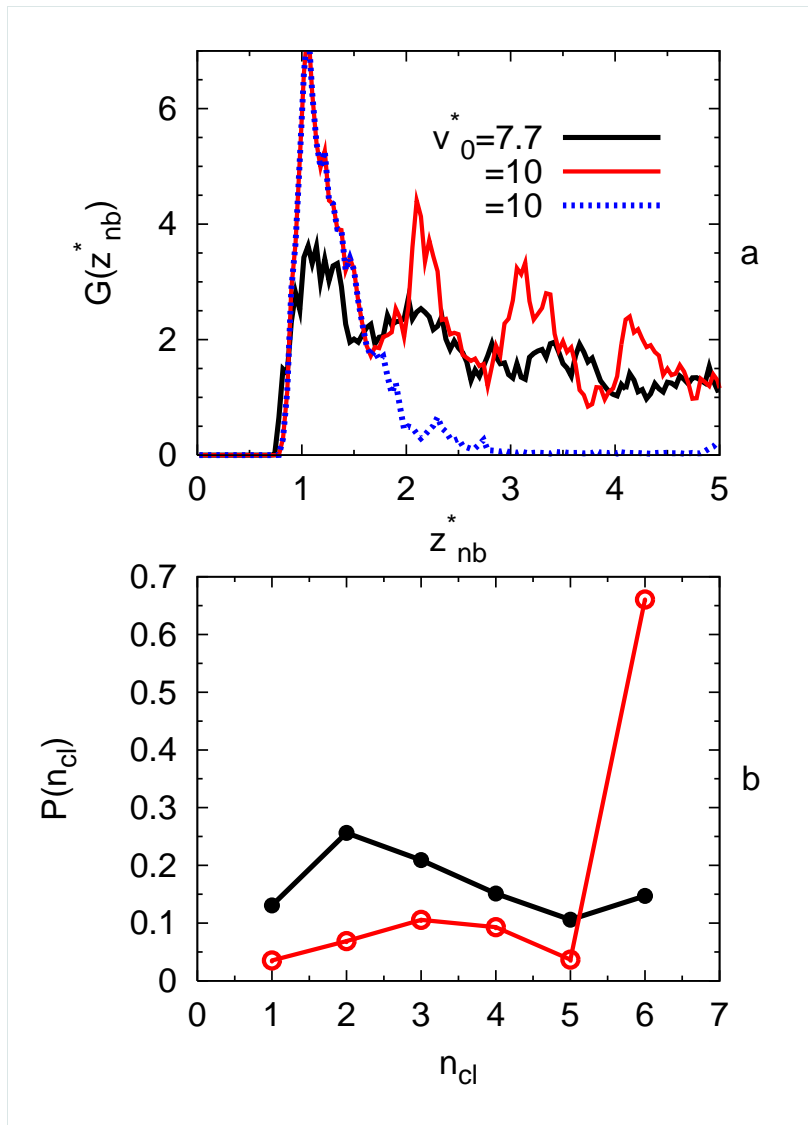


- Single vesicles more deformed  $\rightarrow$  move faster
- Effective hydrodynamic attraction stabilizes clusters

J.L. McWhirter, H. Noguchi, G. Gompper, Proc. Natl. Acad. Sci. 106 (2009)

# RBC Clustering & Alignment in Flow

Low hematocrit  $H_T$ :



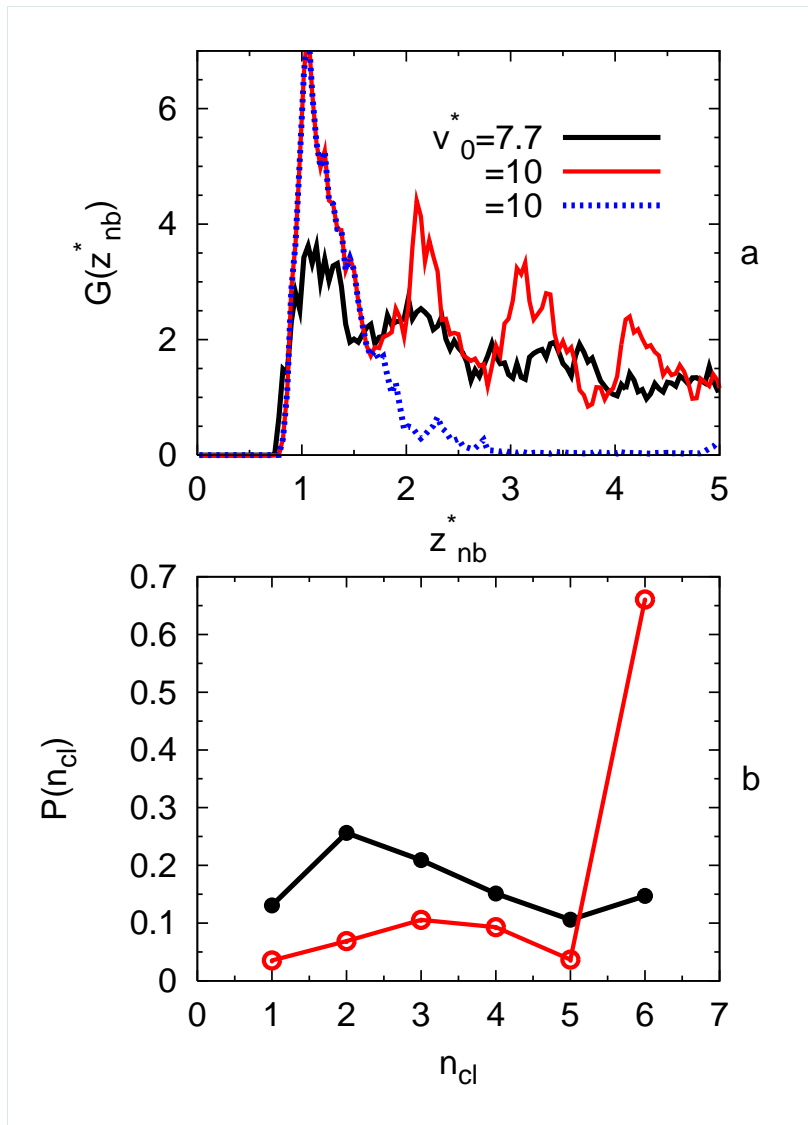
Positional correlation function

Probability for cluster size  $n_{cl}$

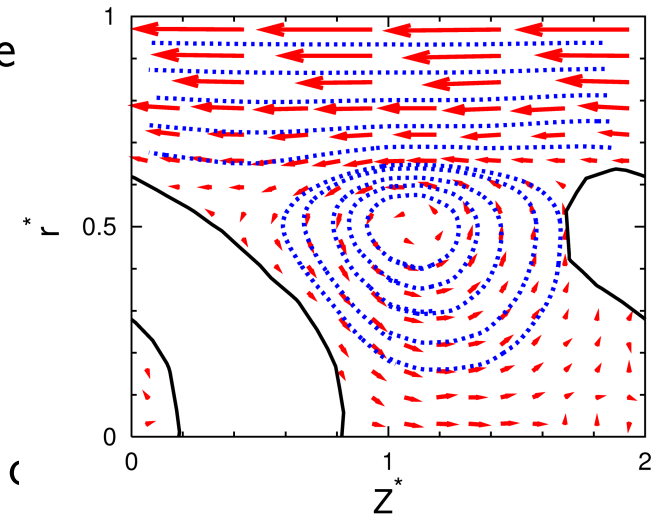
Clustering tendency **increases** with increasing flow velocity

# RBC Clustering & Alignment in Flow

Low hematocrit  $H_T$ :



Positional corre



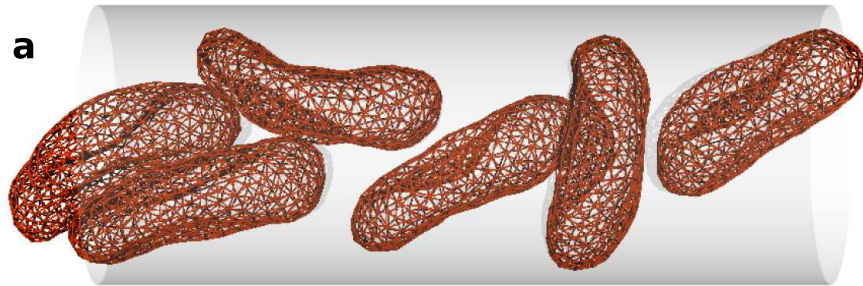
Probability for  $\epsilon$

Clustering tendency **increases** with increasing flow velocity

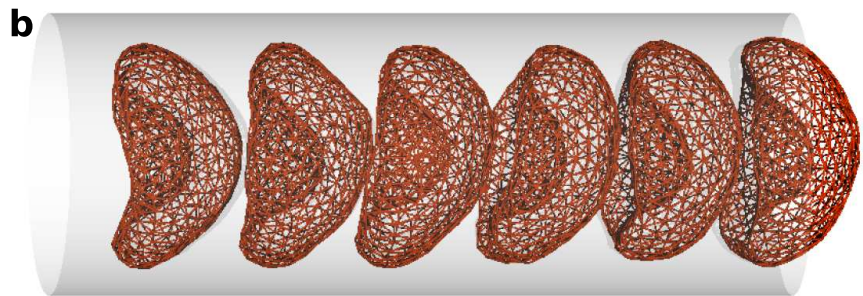


# RBC Clustering & Alignment in Flow

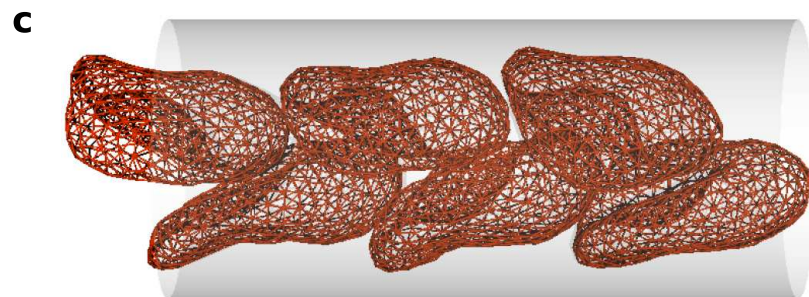
High hematocrit  $H_T$ :



disordered discocyte



aligned parachute



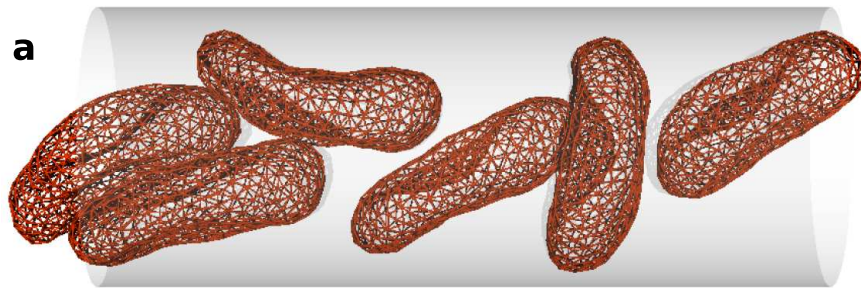
zig-zag slipper



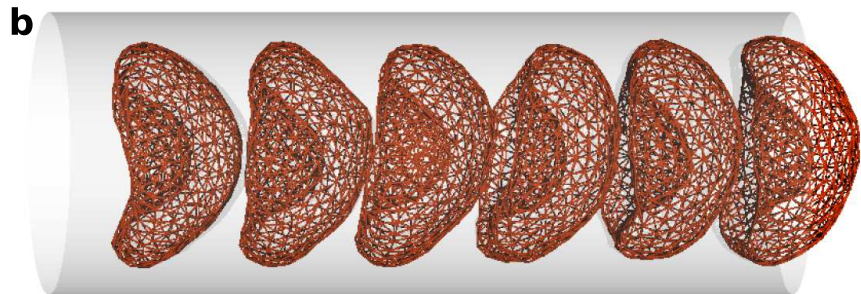
# RBC Clustering & Alignment in Flow

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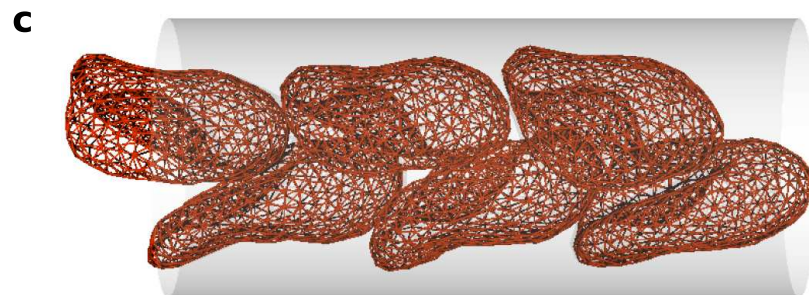
High hematocrit  $H_T$ :



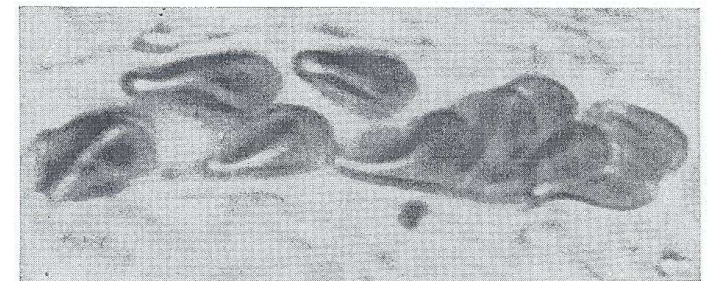
disordered discocyte



aligned parachute



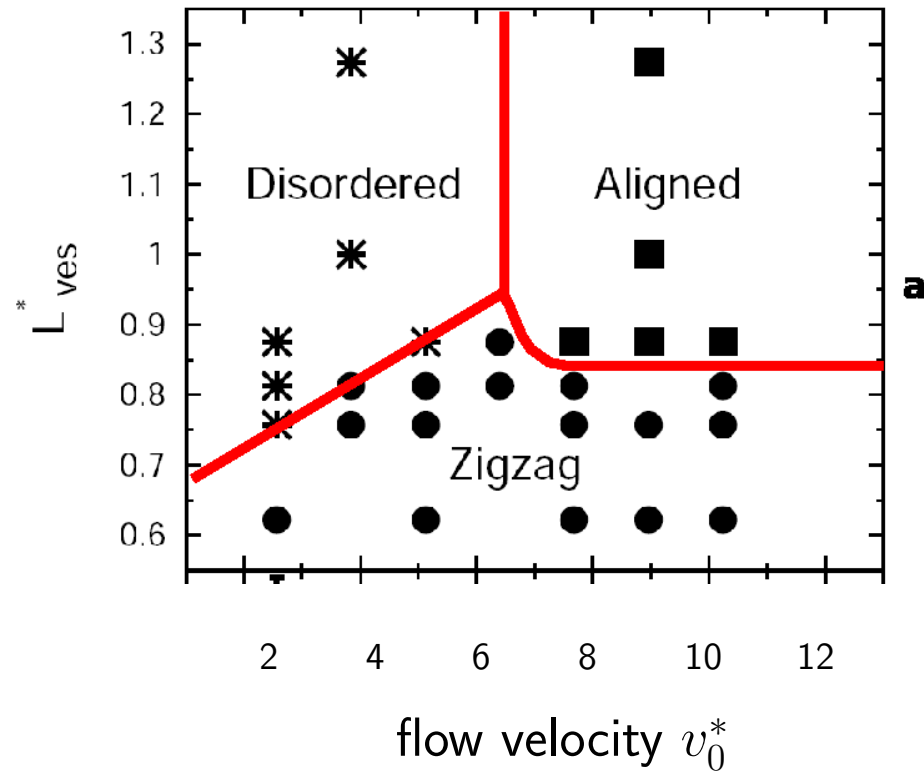
zig-zag



Skalak, Science (1969)

# Clustering & Alignment in Flow

Phase diagram:



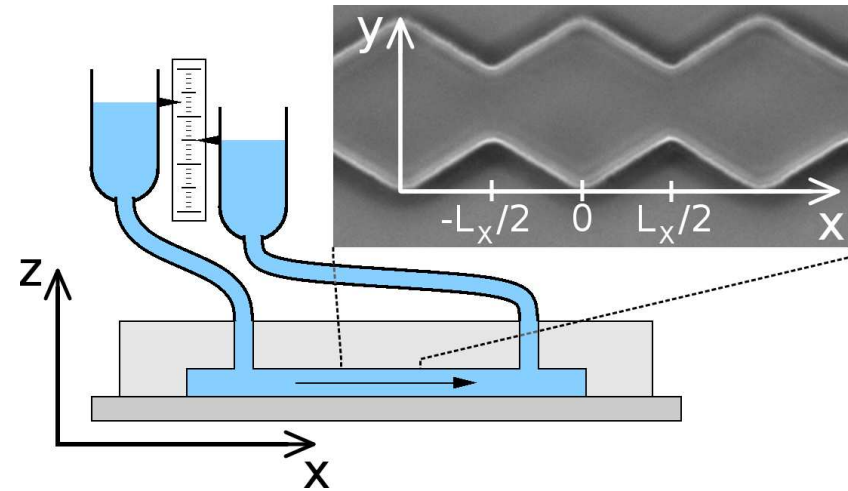
Hematocrit  $H_T = 0.28/L_{ves}^*$

Transition to zig-zag phase despite **higher** flow resistance than aligned-parachute phase!

# Vesicles in Structured Channels

Vesicle motion through zig-zag shaped channel ( $L_x = 100\mu m$ ):

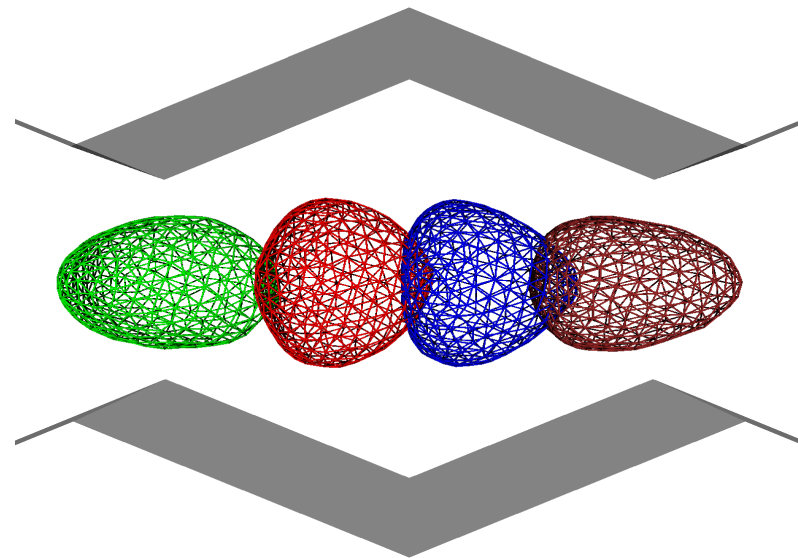
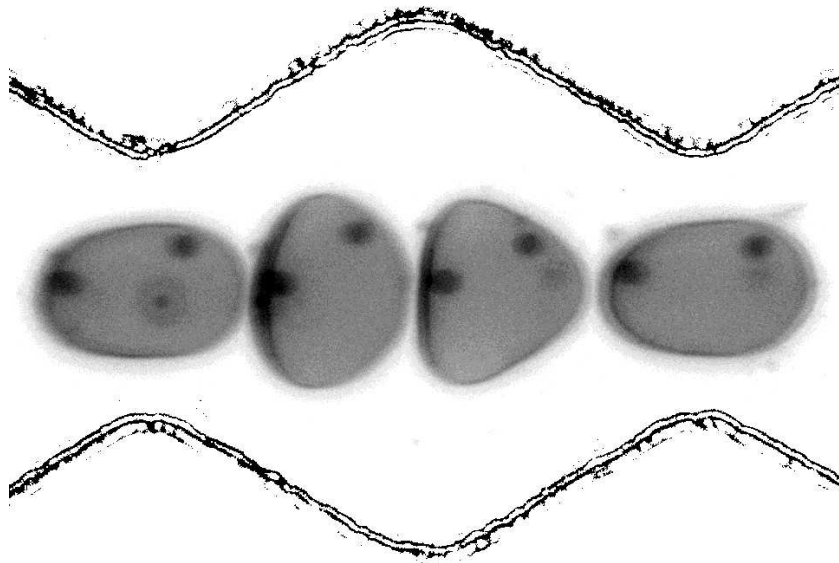
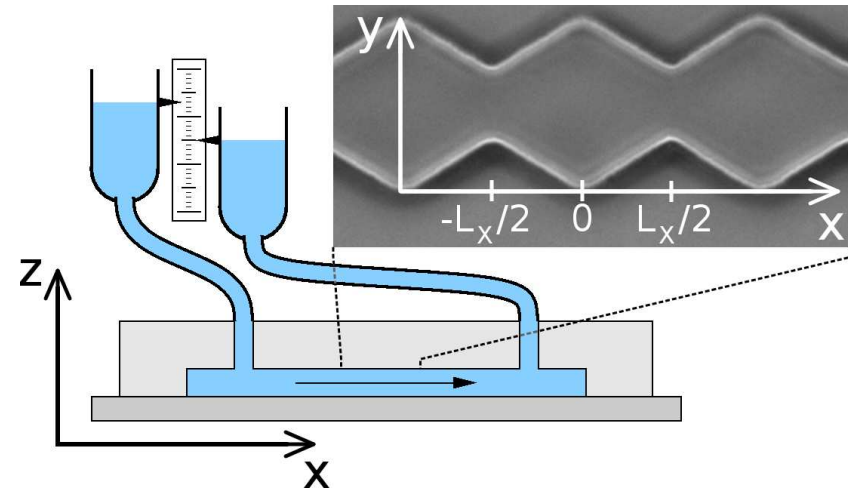
time-dependent flow



# Vesicles in Structured Channels

Vesicle motion through zig-zag shaped channel ( $L_x = 100\mu m$ ):

time-dependent flow



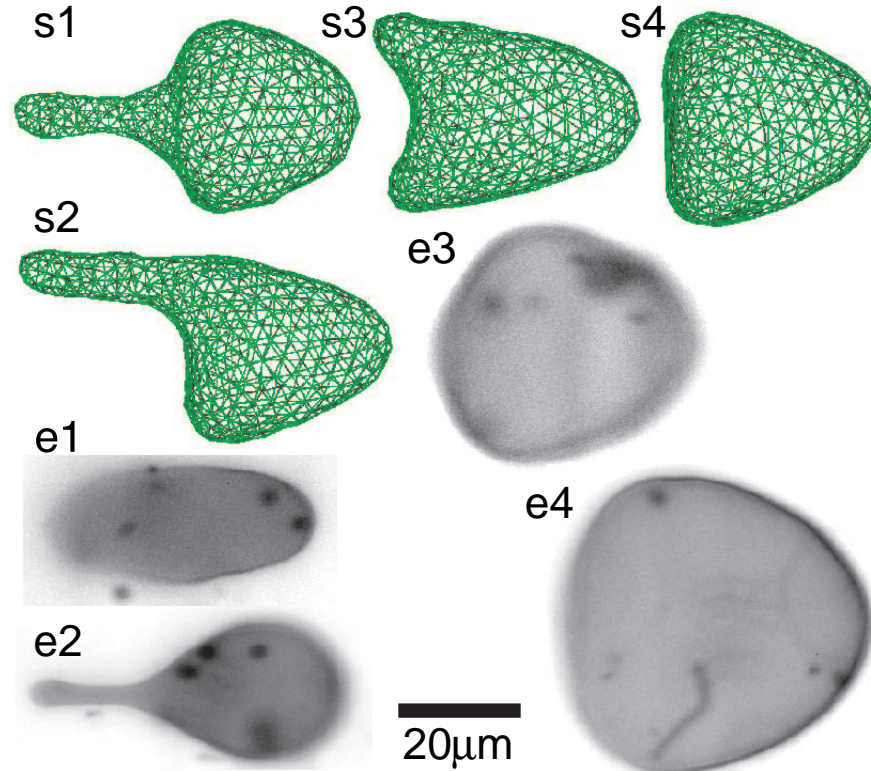
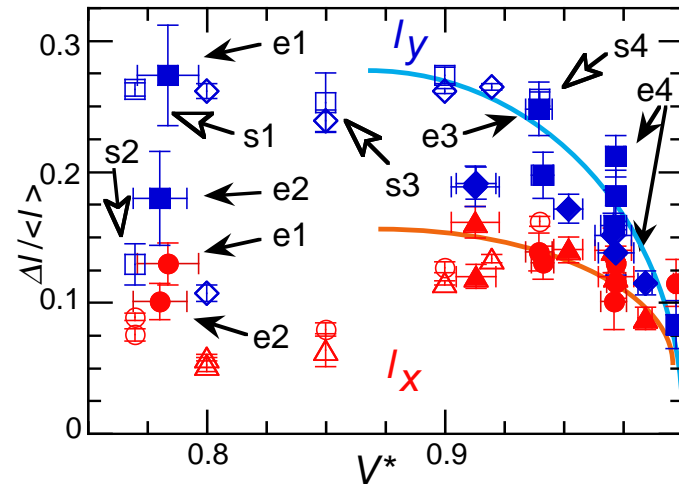
# Vesicles in Structured Channels

Large reduced volume  $V^*$ :

- Fast flows: Symmetric shape oscillations
- Slow flows: Orientational oscillations

Smaller reduced volume  $V^*$ :

- Symmetric double tail
- Asymmetric single tail



# Summary

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- **Mesoscale simulation techniques** are powerful tool to bridge the length- and time-scale gap in complex fluids
- Multi-particle-collision dynamics well suited for hydrodynamics of **embedded particles**: colloids, polymers, vesicles, RBCs
- Vesicles in **shear flow**: tank-treading, tumbling, swinging, lift force
- Red blood cells in **capillary flow**: shear elasticity implies parachute shapes, hydrodynamic clustering and alignment
- Vesicles in **structured channels**: single- and double-tailed shapes

**Review**: G. Gompper, T. Ihle, D.M. Kroll, R.G. Winkler, Adv. Polym. Sci. **221**, 1 (2009)